What can be decided locally without identifiers?

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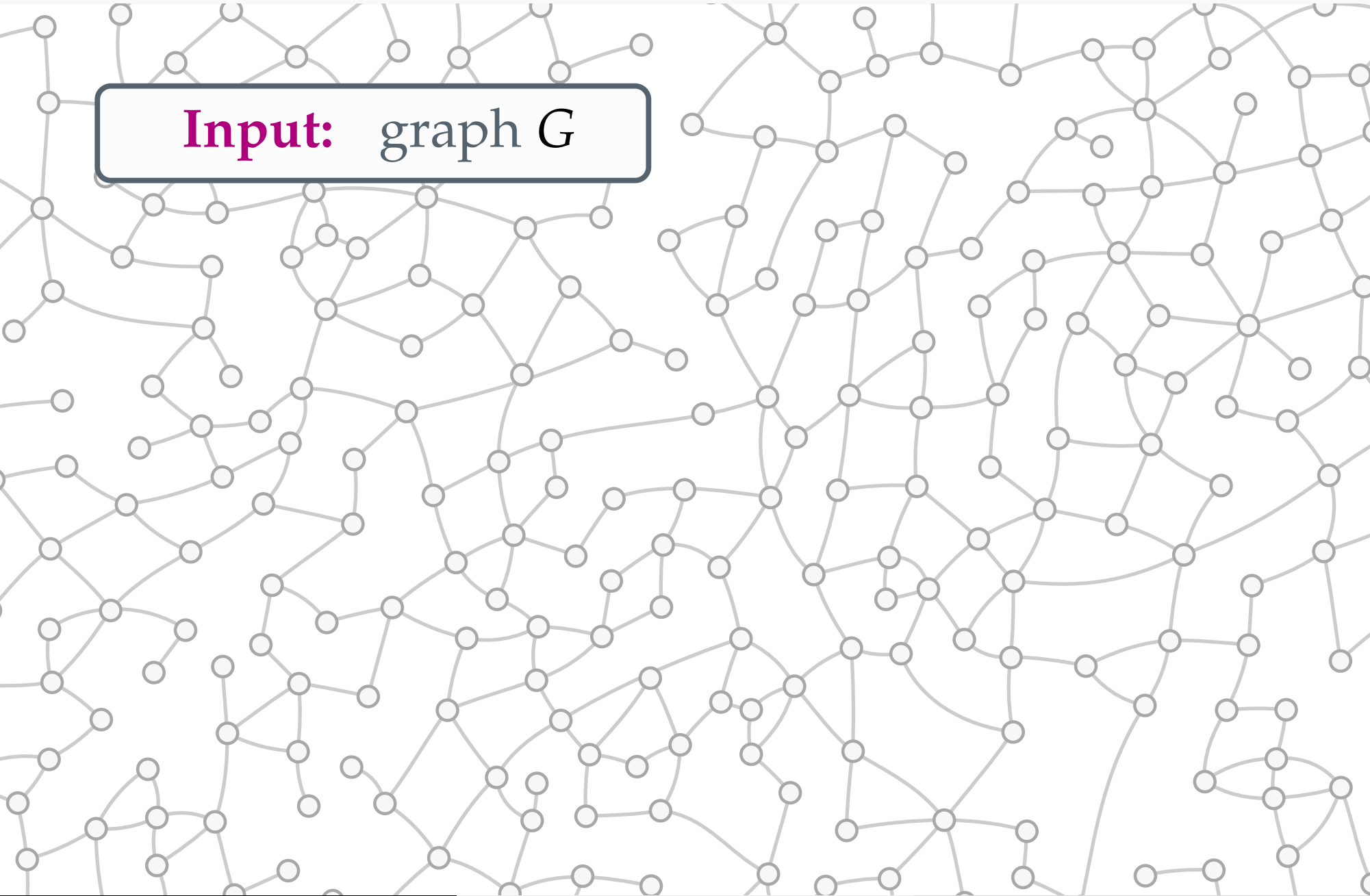
Local decision

[FKP **FOCS'11**]



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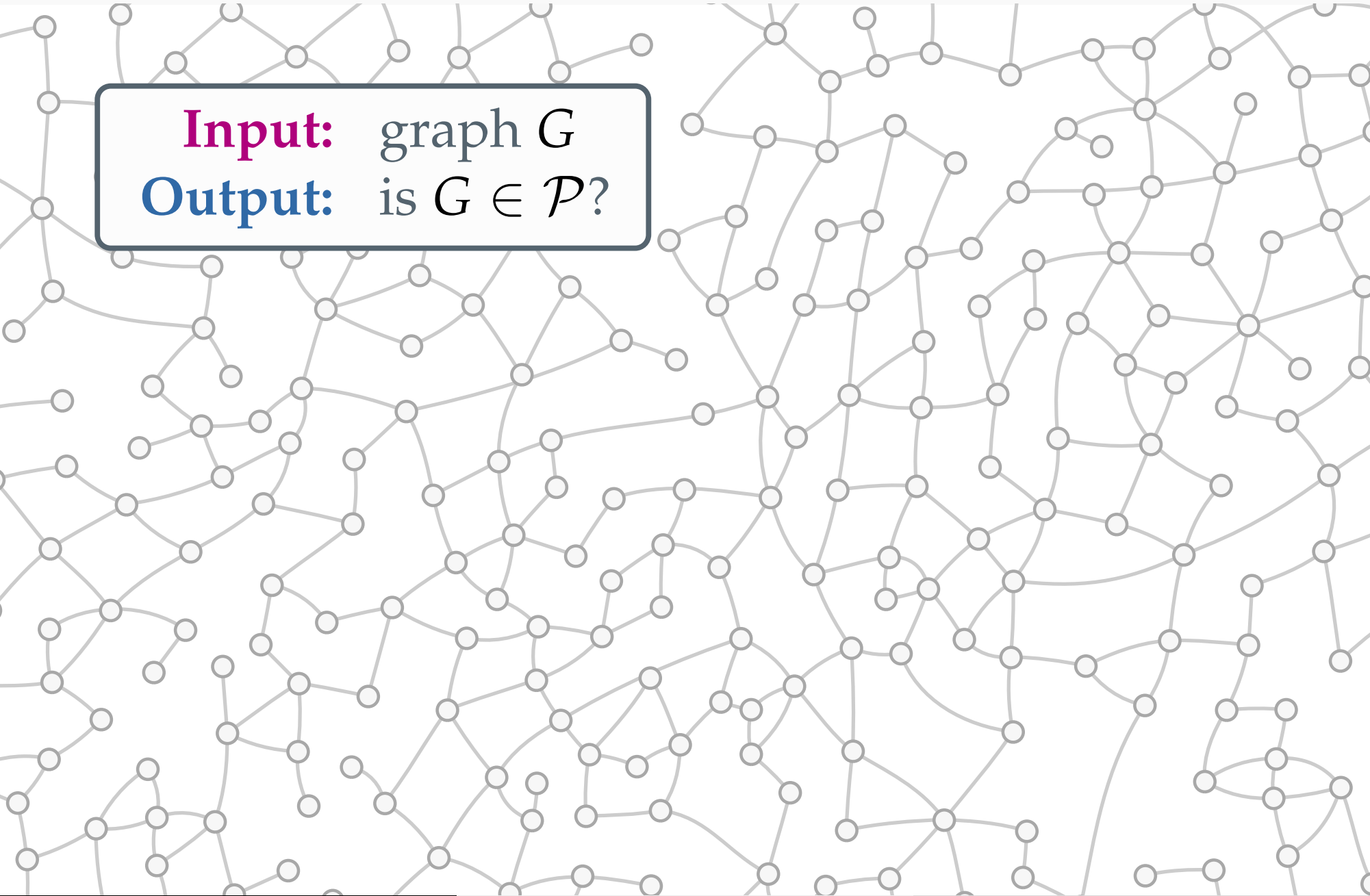

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Output: is $G \in \mathcal{P}$?

Local decision

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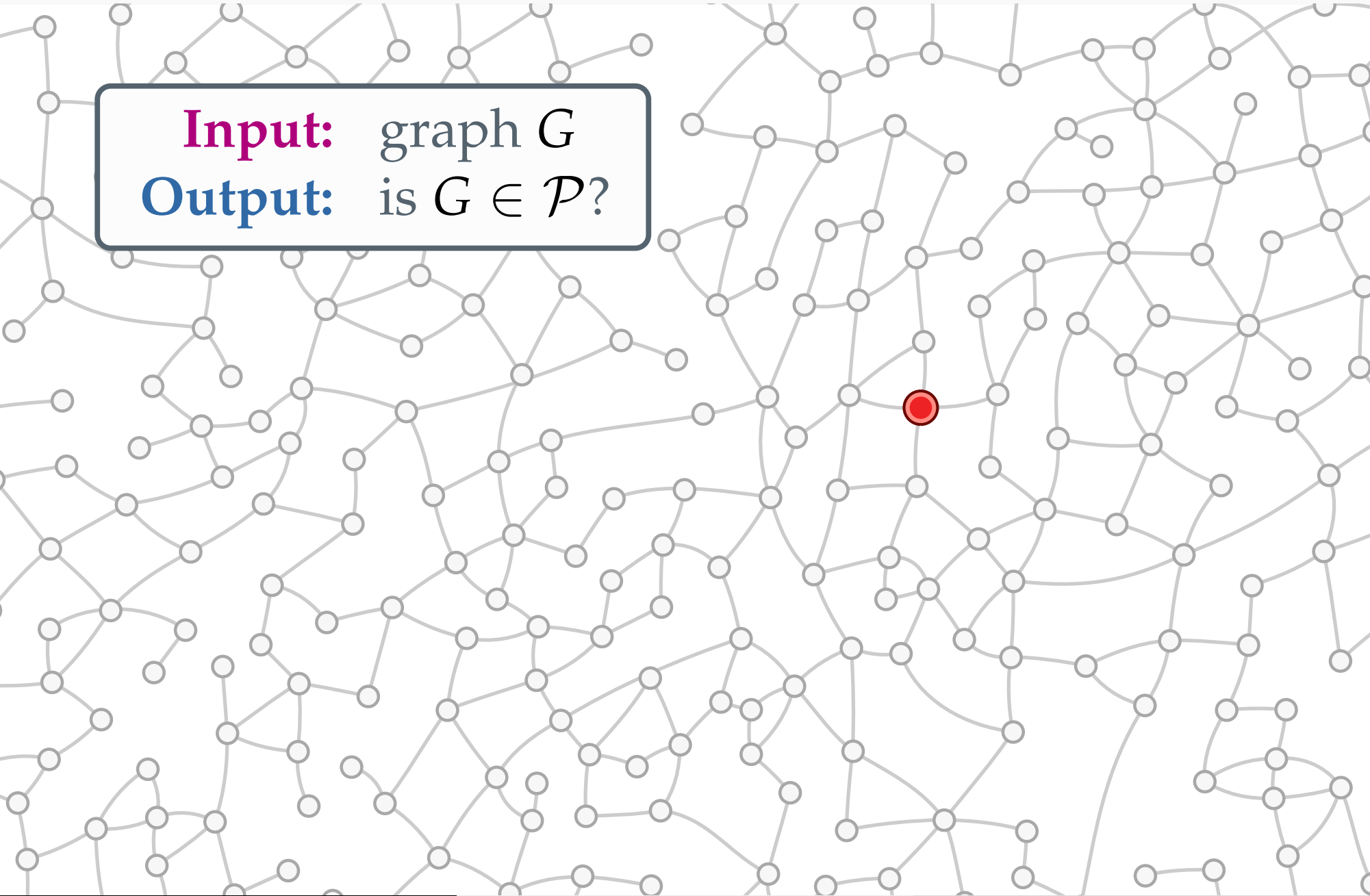

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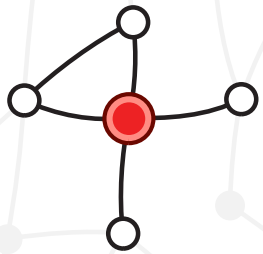

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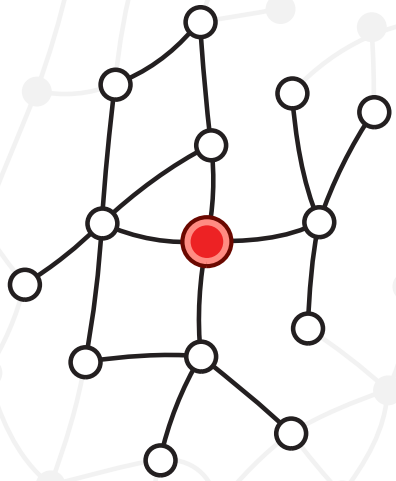

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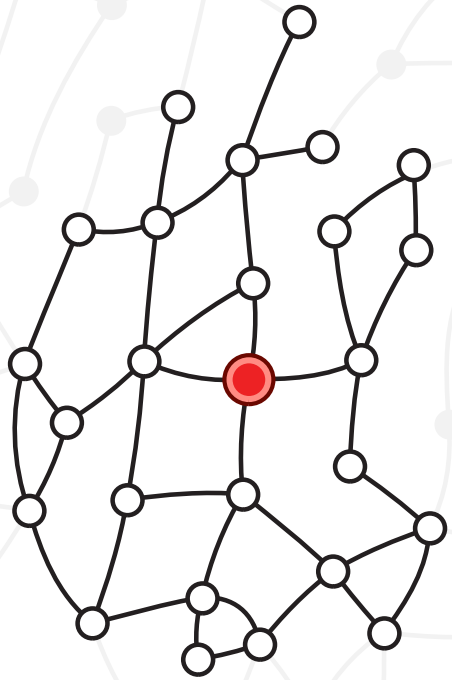

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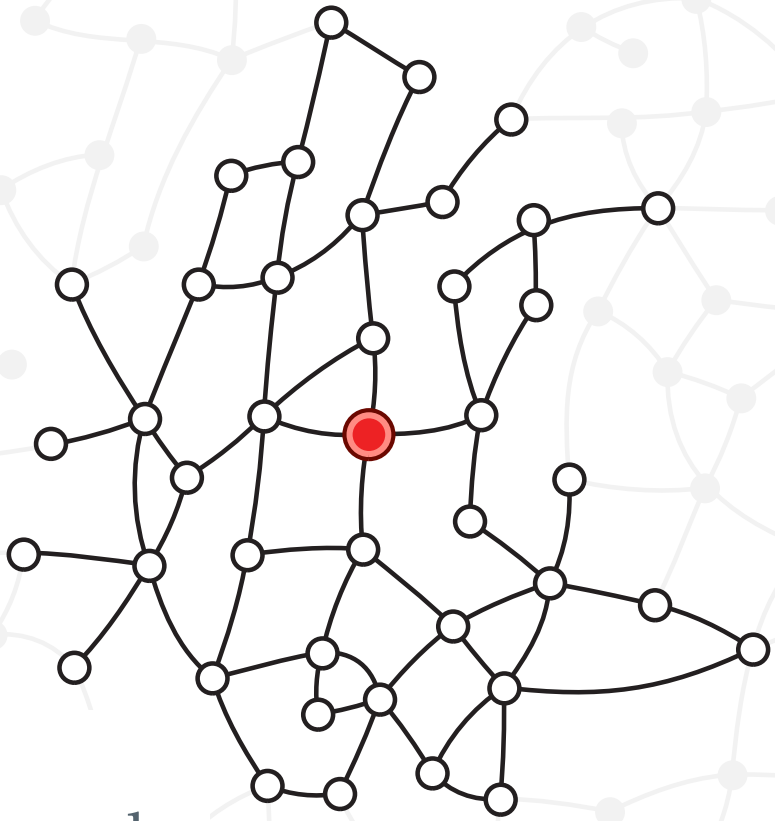

Local algorithm
$\equiv$ $O(1)$ communication rounds
$\equiv$ $O(1)$ radius neighbourhood

Local decision

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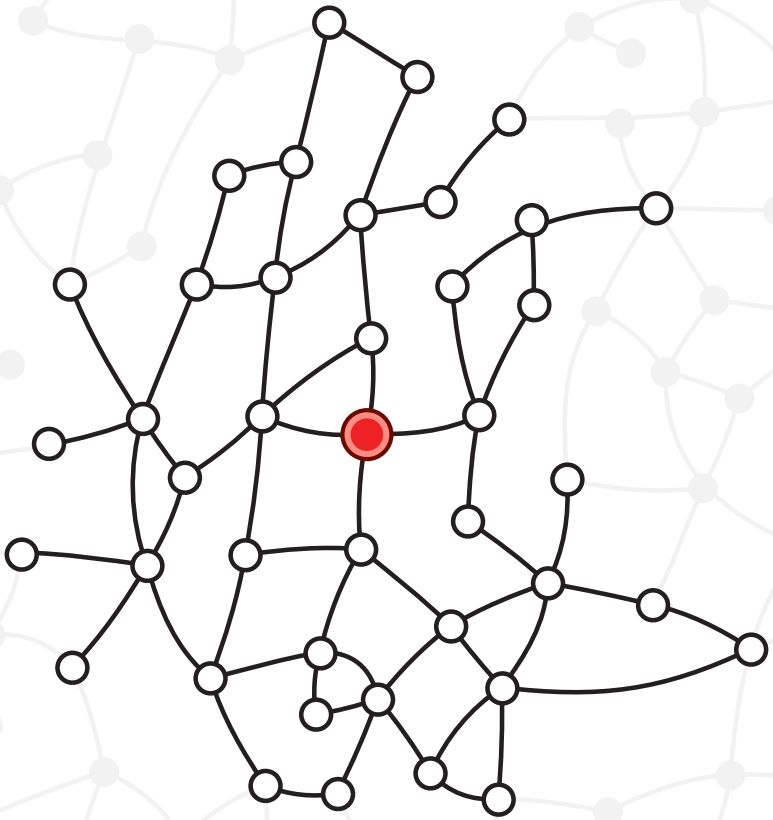

yes / *no*

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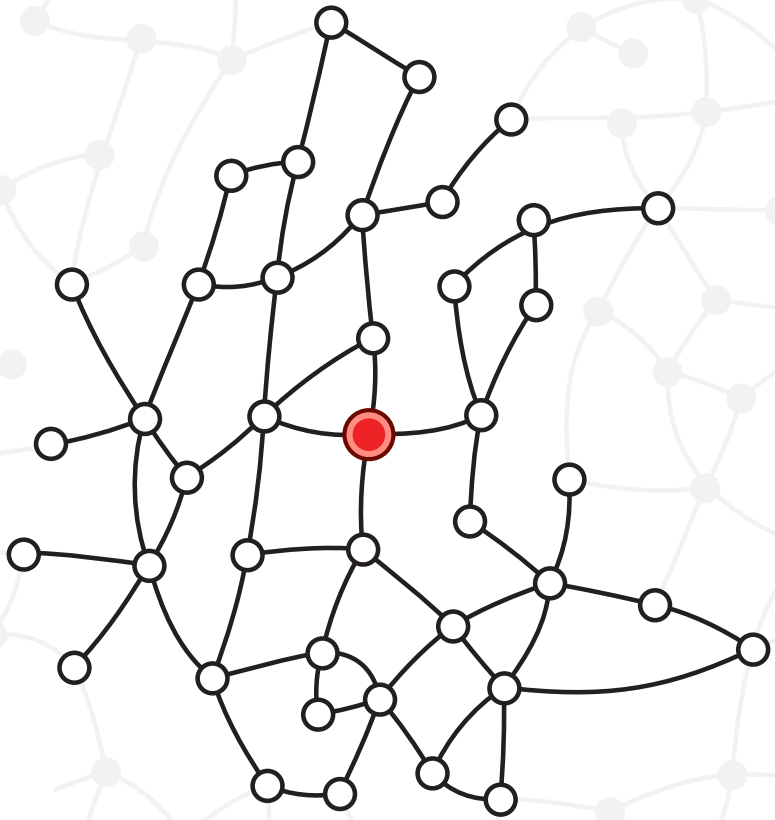

***G* is accepted iff**
all nodes ouput ***yes***

Local decision

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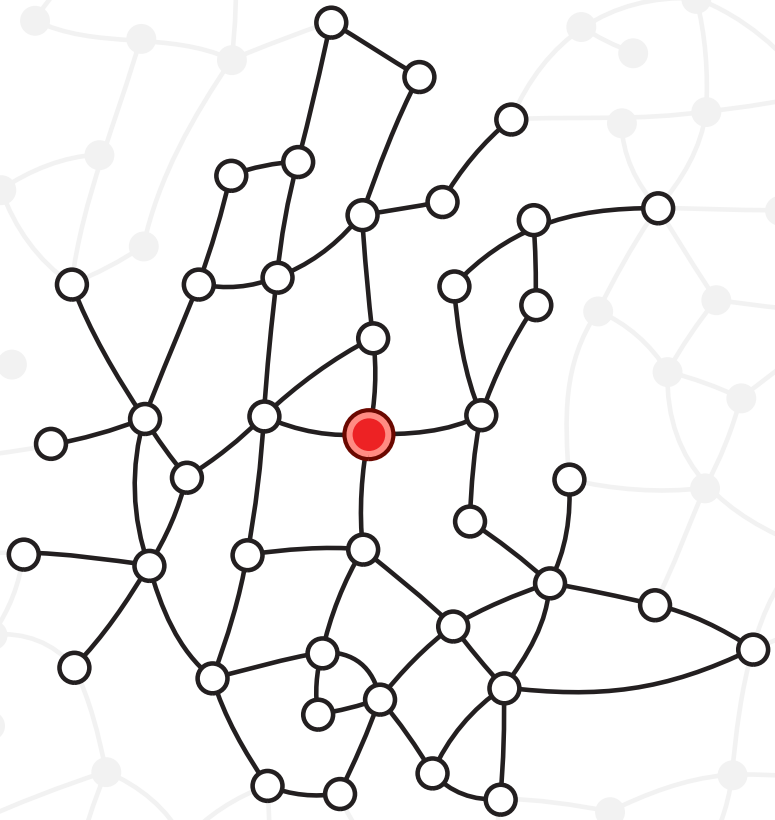
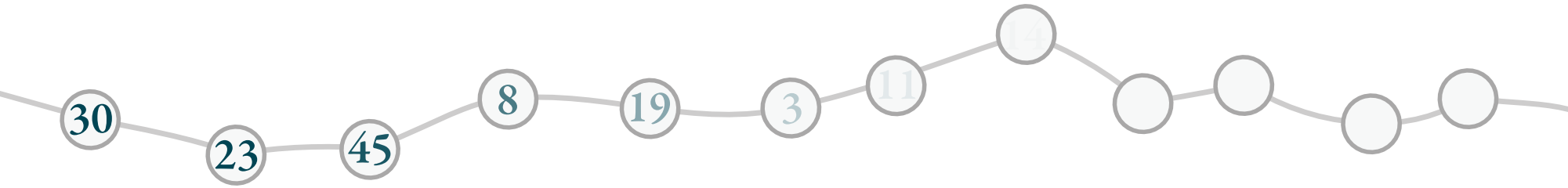


Locally decidable $\mathcal{P}$:

- triangle-freeness
- Eulerian graphs
- line graphs
- Locally checkable labellings (G, ℓ)

Our question



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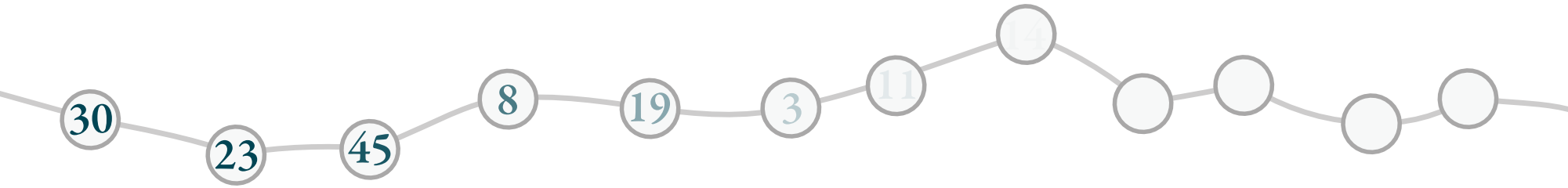


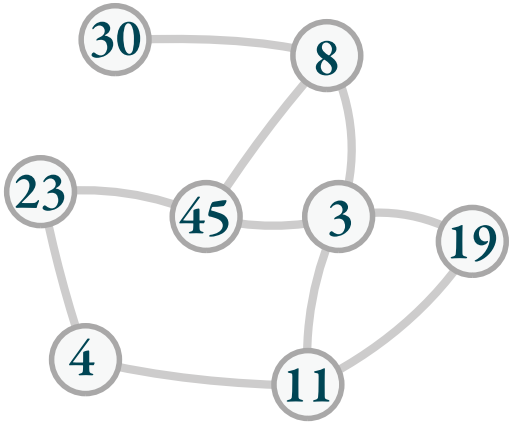
We ask: Do **node identifiers** help in local decision?

IDs do not seem useful...

- Graph properties do not depend on node labels
- **Symmetry breaking** is not needed for decision problems!

Our question—formalised

[FHK **OPODIS'12**]



LOCAL model
(deterministic)

$V(G) \subseteq \{1, 2, 3, \ldots\}$

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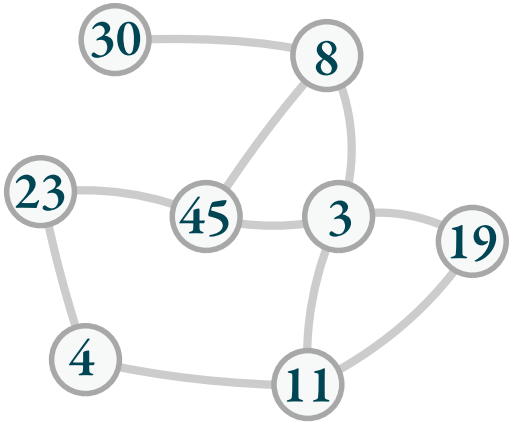
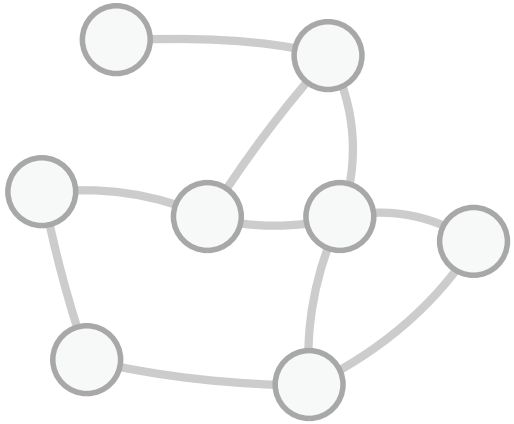


VS.



$\mathcal{LOCAL}$ model

(deterministic)

$V(G) \subseteq \{1, 2, 3, \ldots\}$

ID-oblivious model

Restriction: Output is **invariant under relabelling** the nodes

(i.e., depends only on **topology**)

Easy cases

Warm up!

Under some assumptions:

$$\mathcal{LOCAL} = \text{ID-oblivious}$$

Proof by simulation...

Easy cases

Let A be a $\mathcal{LOCAL}$ decision algorithm

ID-oblivious simulation of A

Input: local neighbourhood (H, v) of G

For each ID-assignment $f : V(H) \to \{1, 2, \ldots, n\}$:

- if $A(f(H, v)) = no$ then **output** no.

Otherwise **output** yes.

Assumptions:

- Nodes know n

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Main theorem*

$$\mathcal{LOCAL} \neq \text{ID-oblivious}$$

(I.e., there is a locally decidable property
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Proof...

Separation under promise

Promise problem

Input:
- $G = (G, M)$ is a labelled $\boldsymbol{n}$-cycle
- M is a Turing machine

Promise:
- If M halts in s steps, then $\boldsymbol{n} \geq s$

Output:
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ID-oblivious **Impossible:** Must solve the Halting Problem

$\mathcal{LOCAL}$ **Possible:** Node v simulates M for $\mathrm{ID}(v)$ steps

Getting rid of the promise

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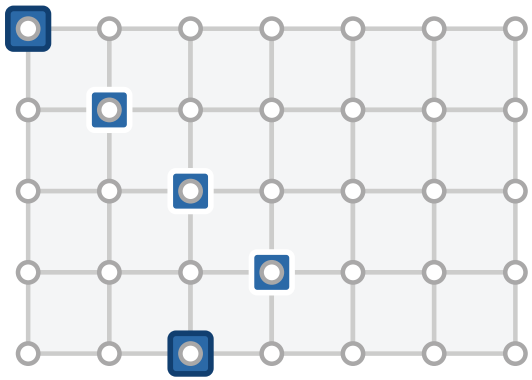
⇓ **Replace!** ⇓



Computation table of M

$\subseteq$ G

yes instance

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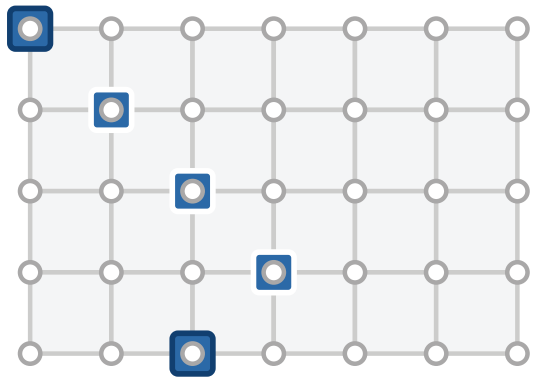

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Interesting bit: Table needs to be **obfuscated!**

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For local decision, we proved:

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	IDs help	IDs don't help
Decision	**This work**	[FHK **OPODIS'12**]
Search	[HHRS **SIROCCO'12**]	[NS **Sicomp'95**] [GHS **PODC'12**]

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For local decision, we proved:

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Randomisation?

- Open problems in **randomised** decision [FKPP **DISC'12**]

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Cheers!