Succinct Arguments: Constructions and Open Problems

Alessandro Chiesa
UC Berkeley

The Role of Cryptography

The **Power** of PCPs

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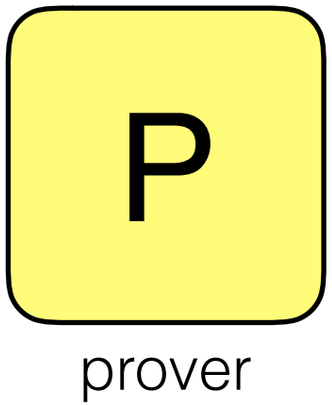

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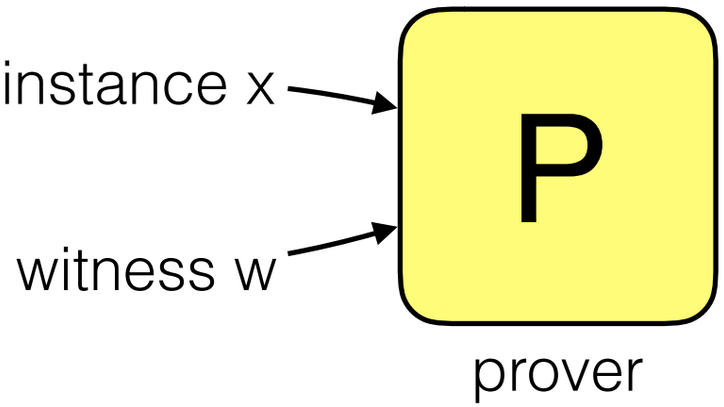

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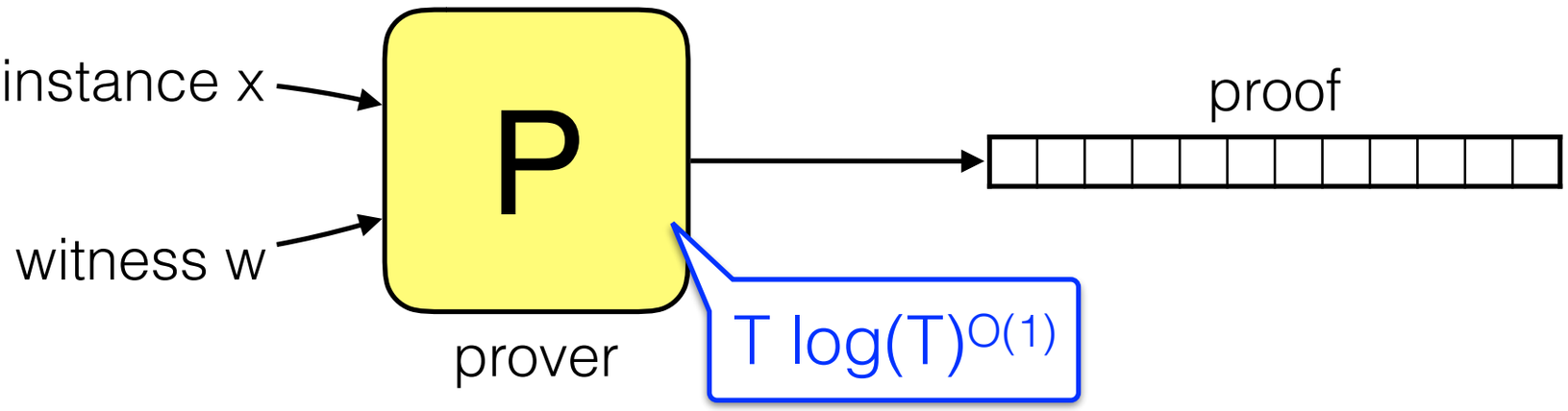

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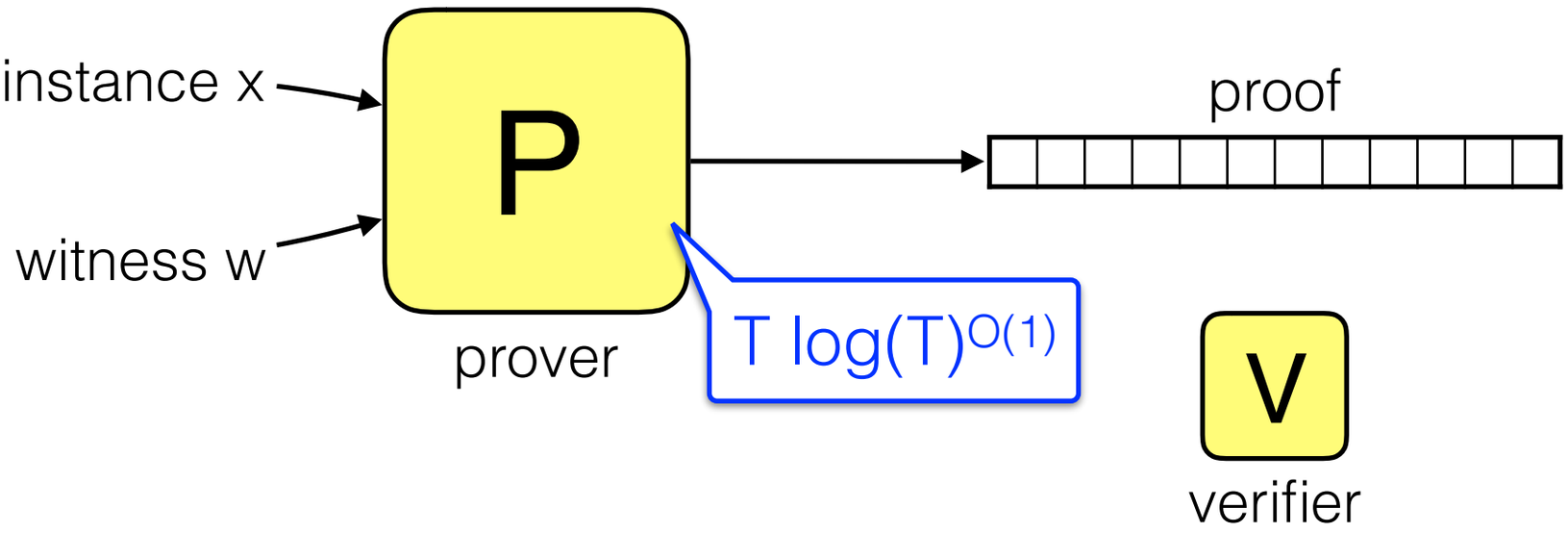

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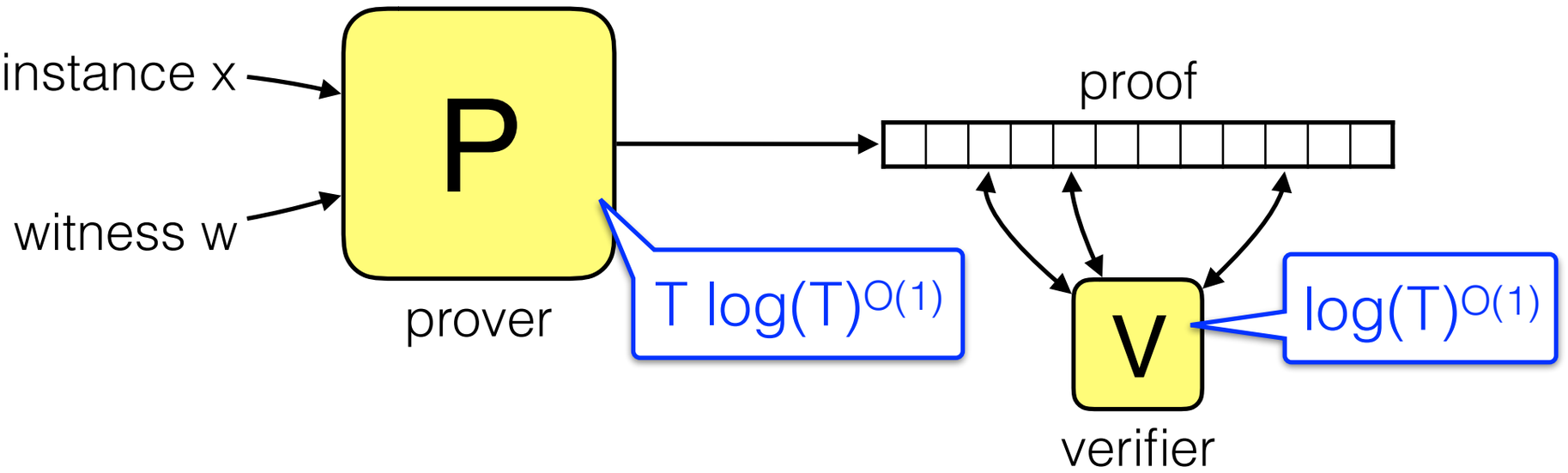

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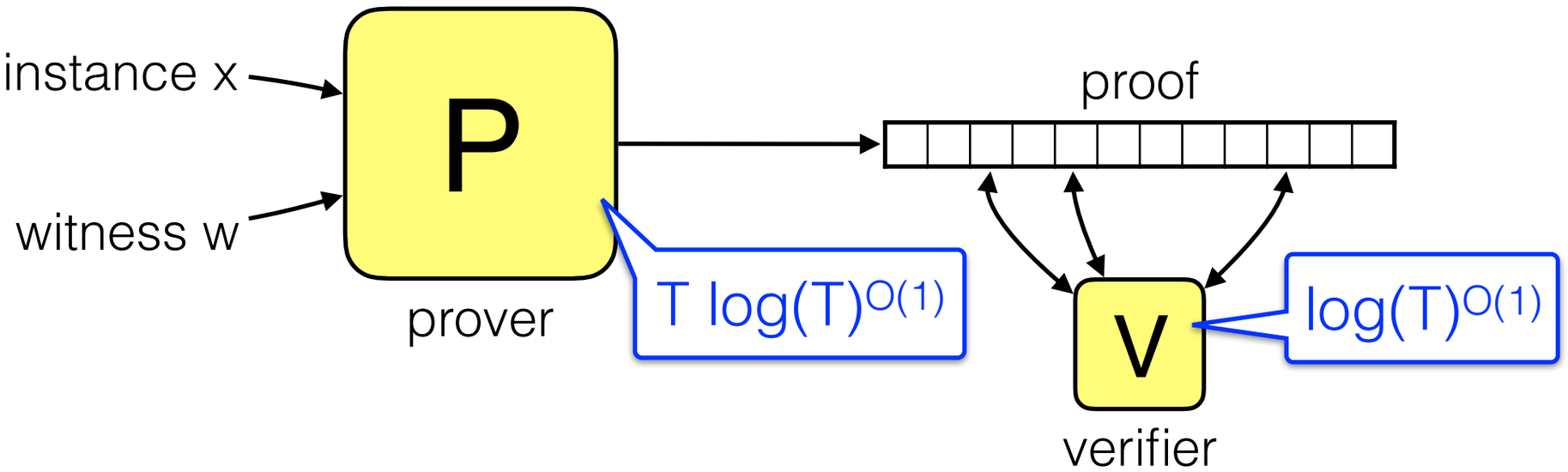

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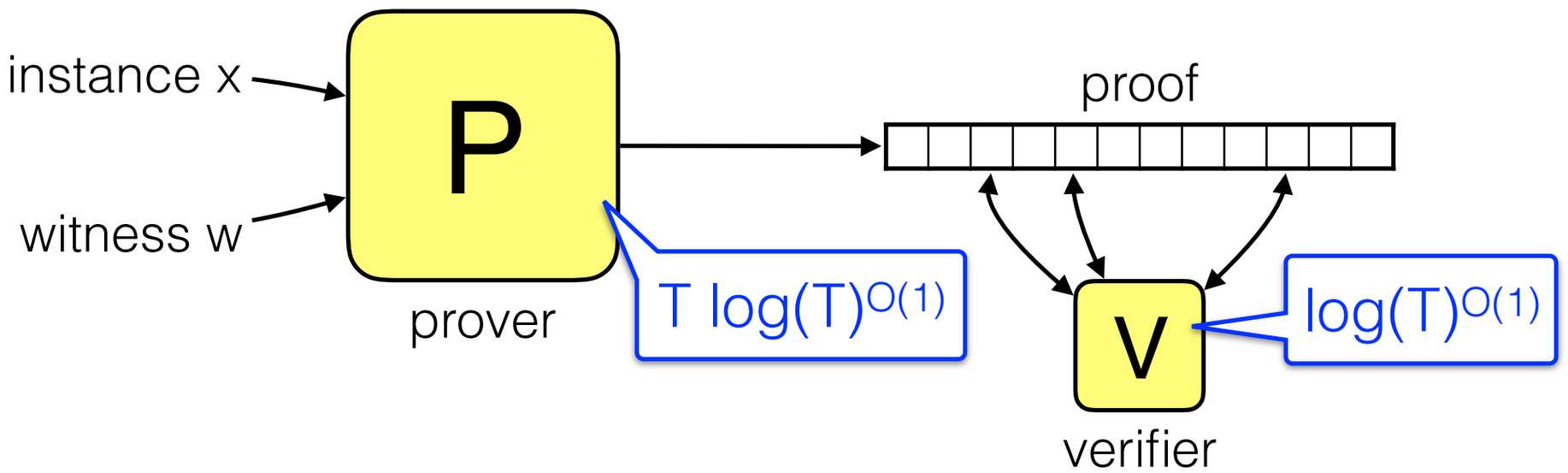

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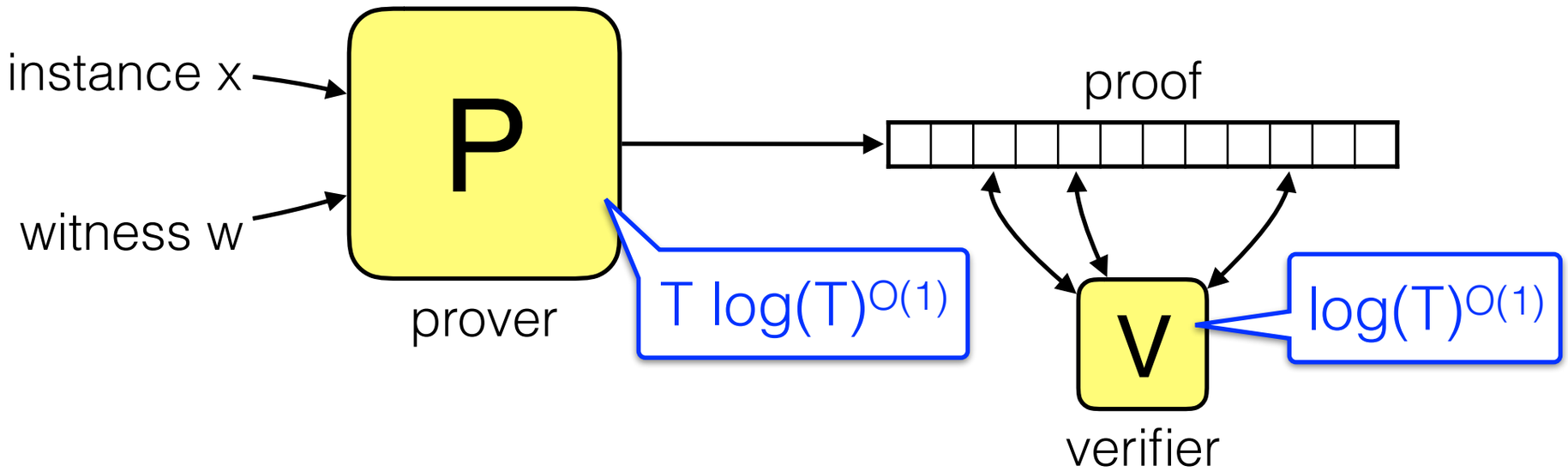

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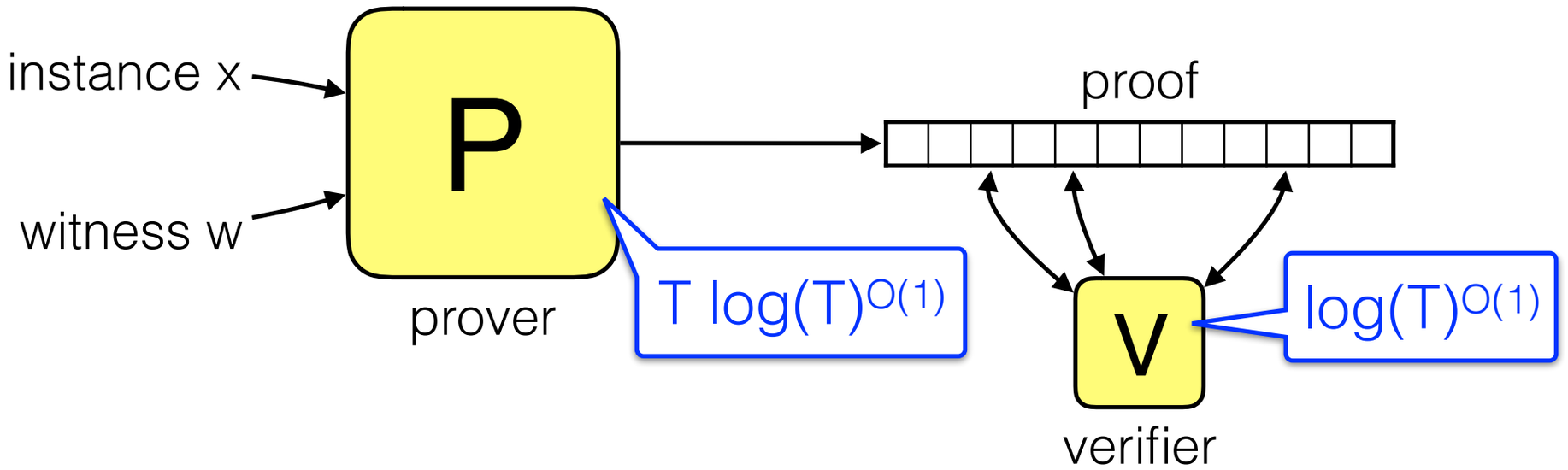

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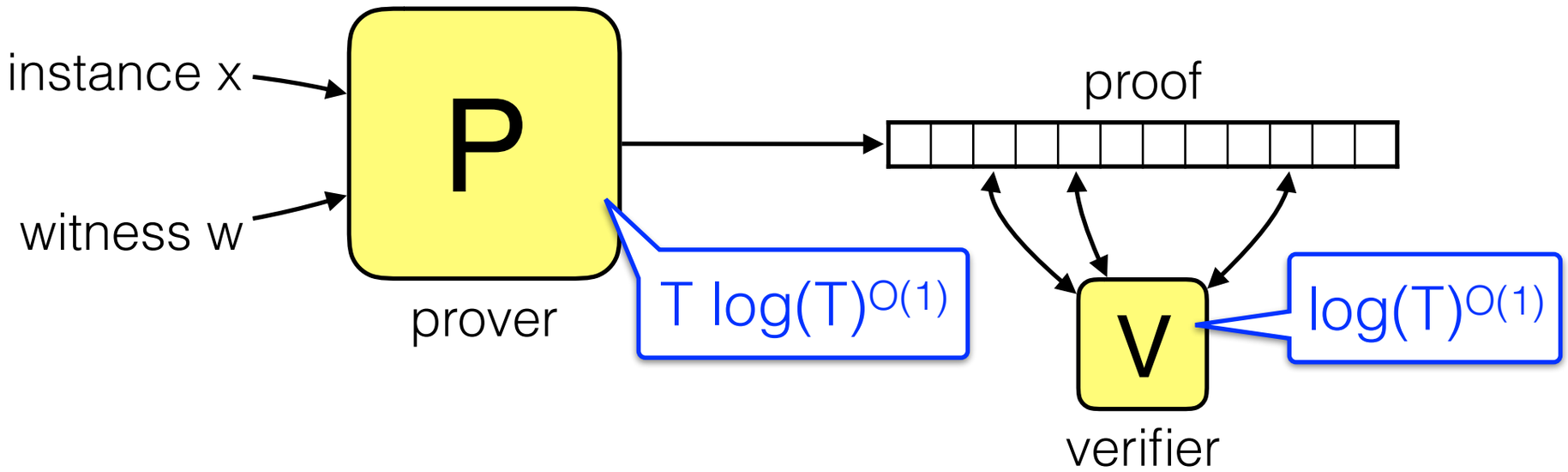

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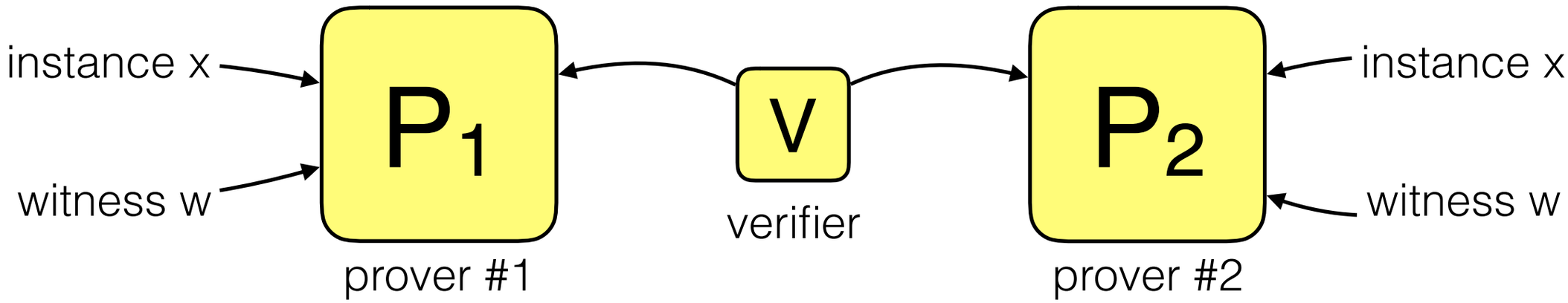

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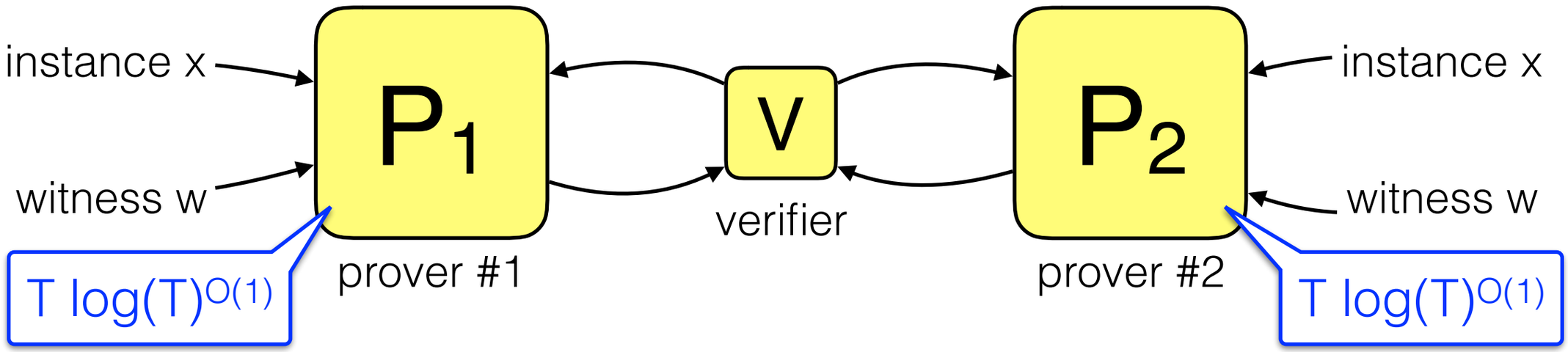

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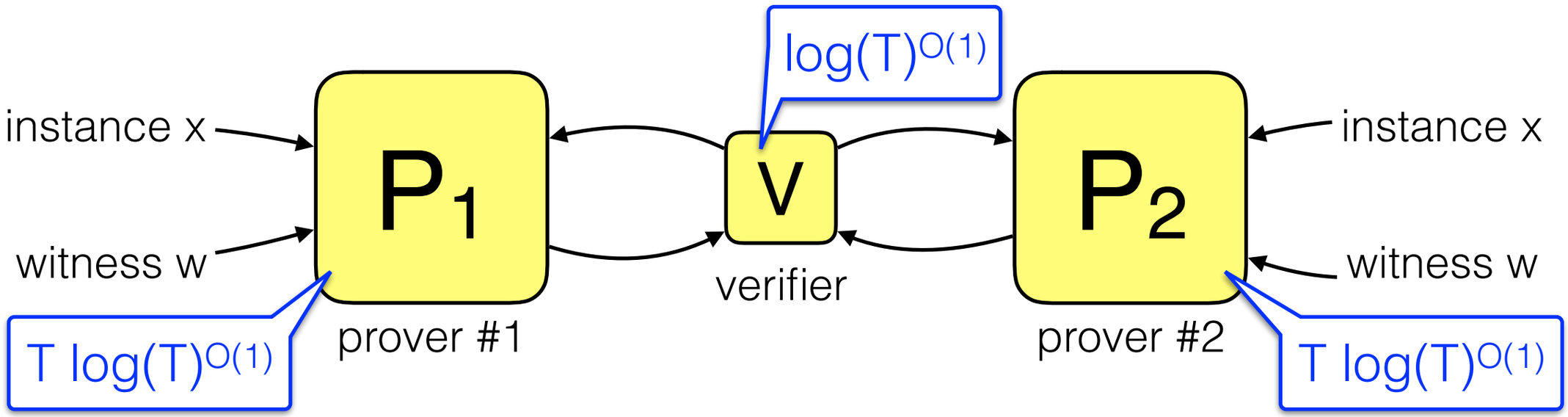

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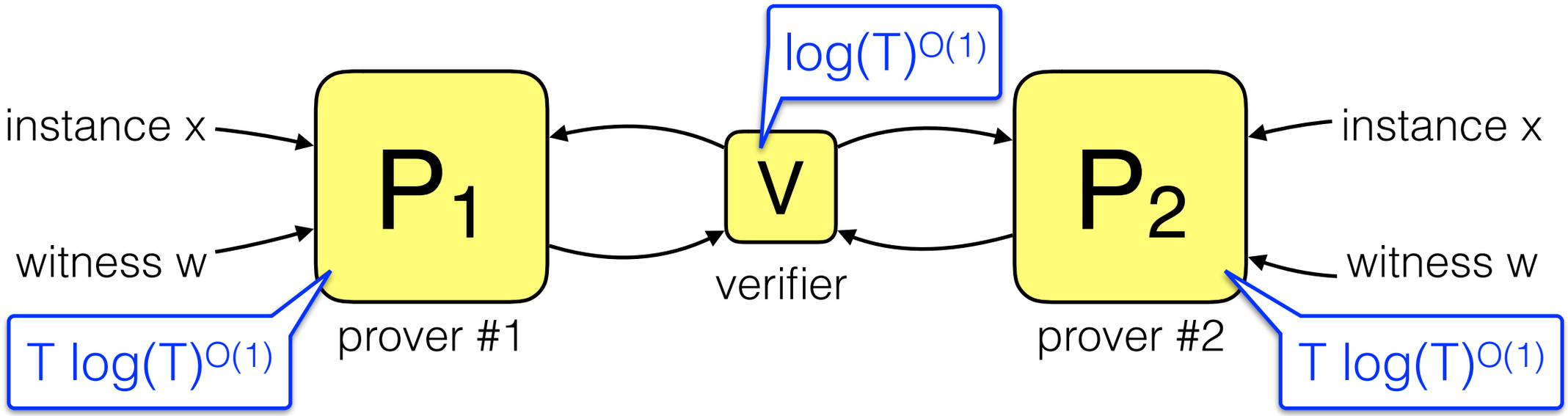

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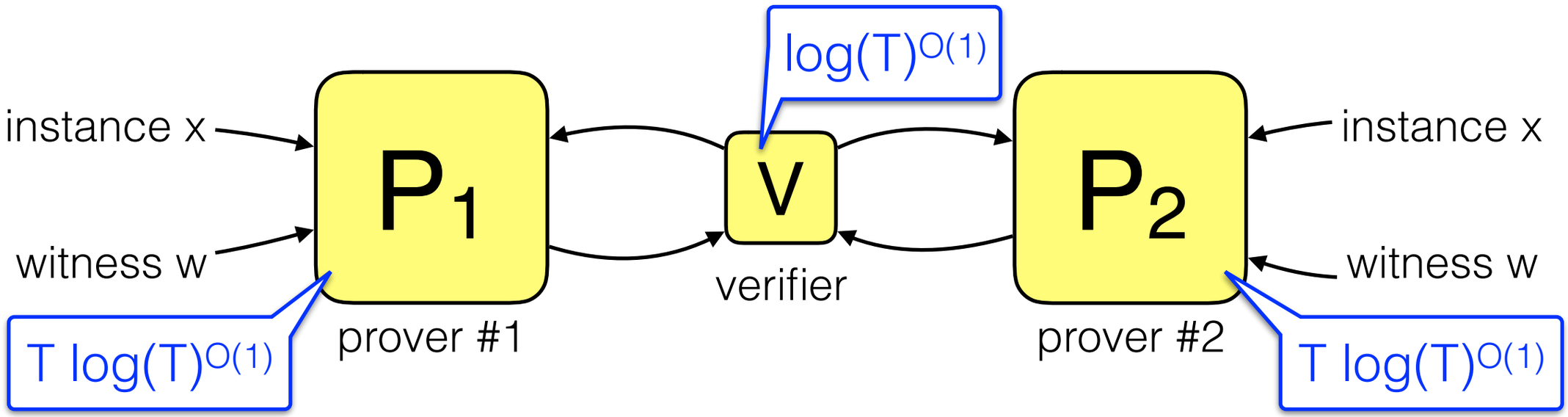

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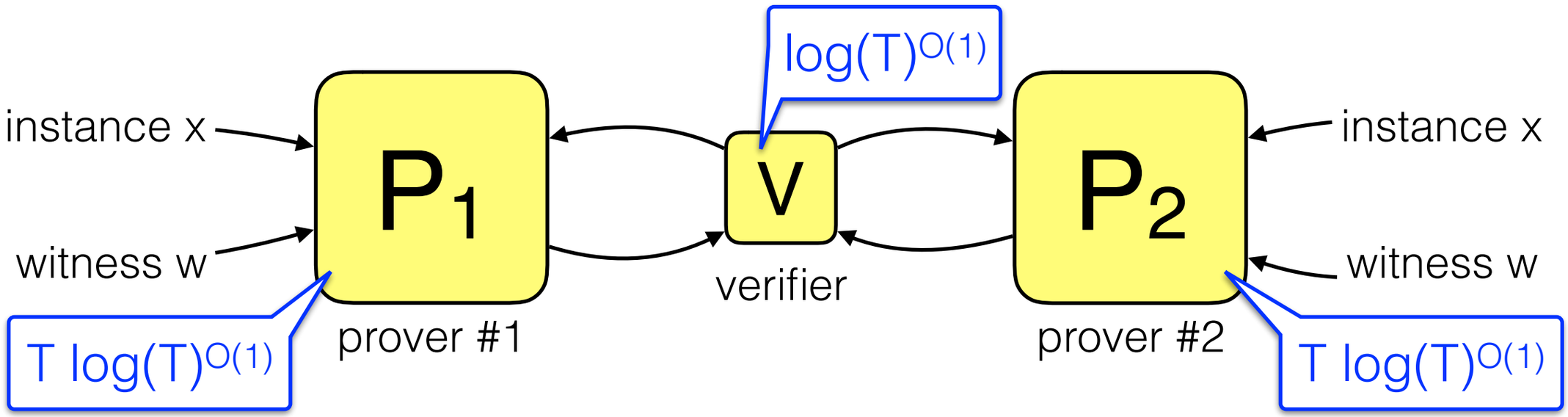

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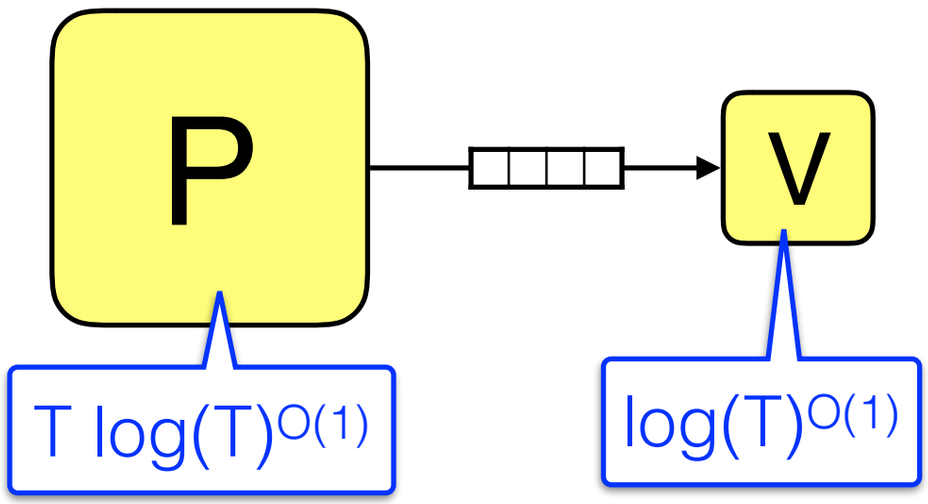

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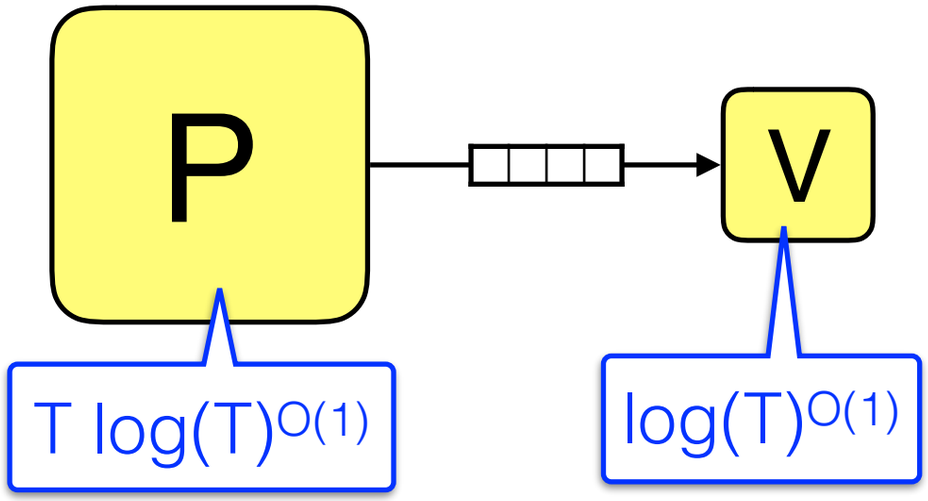

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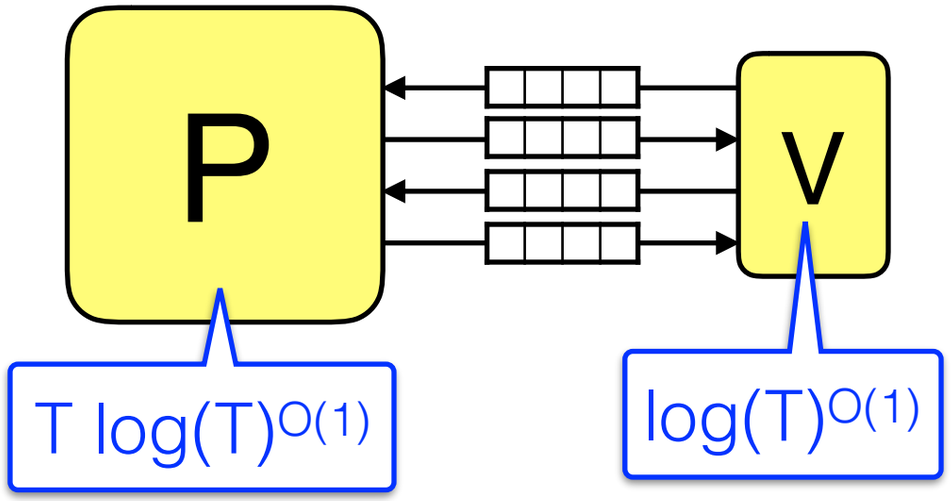

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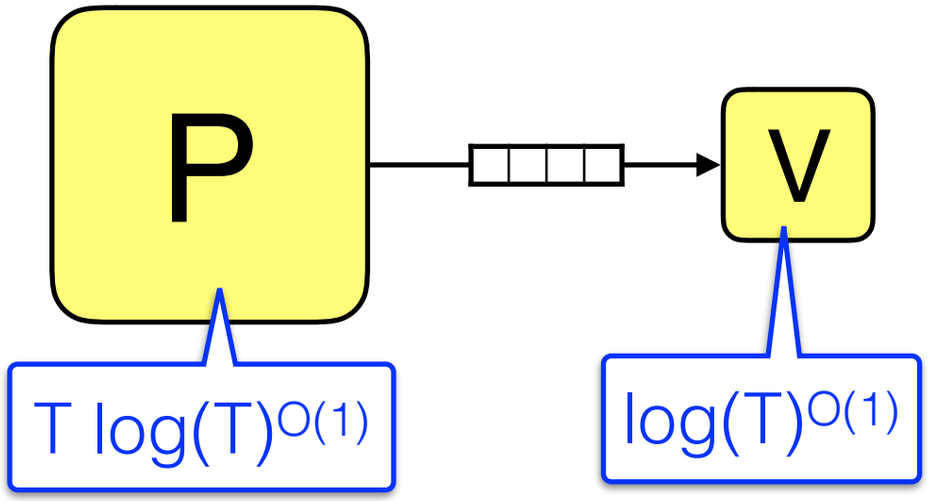

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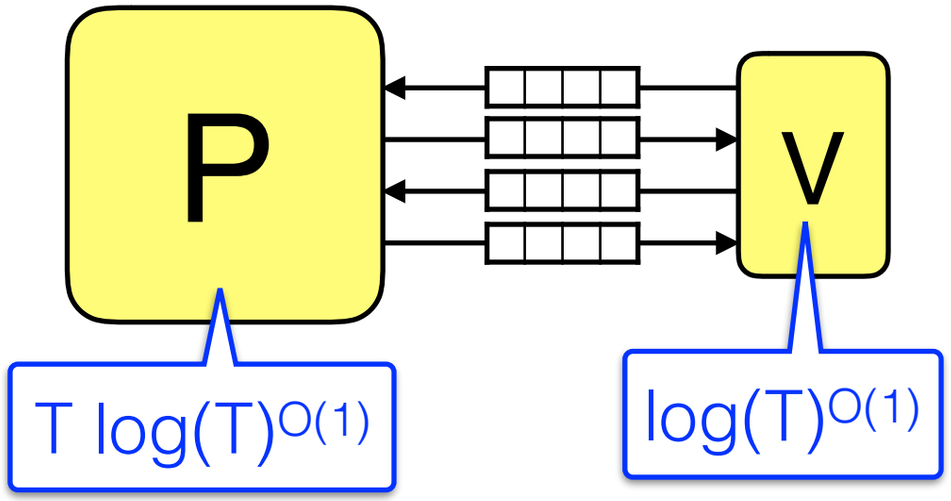

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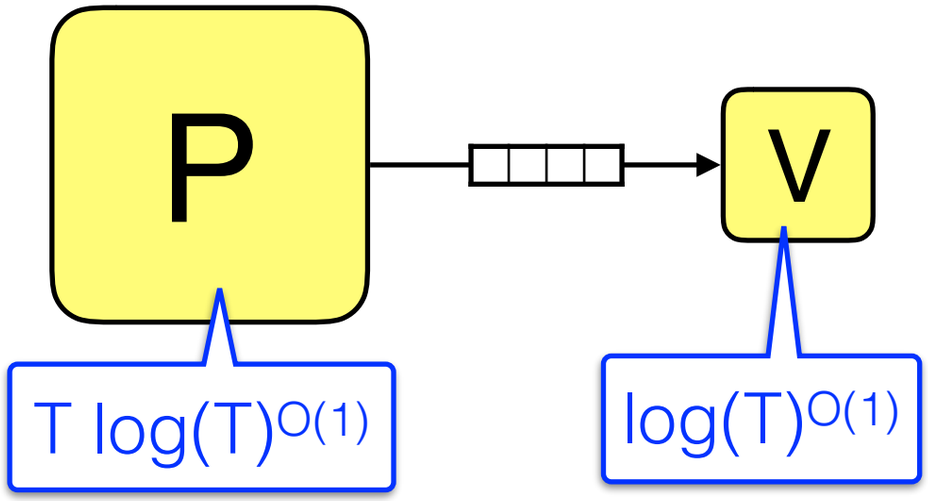

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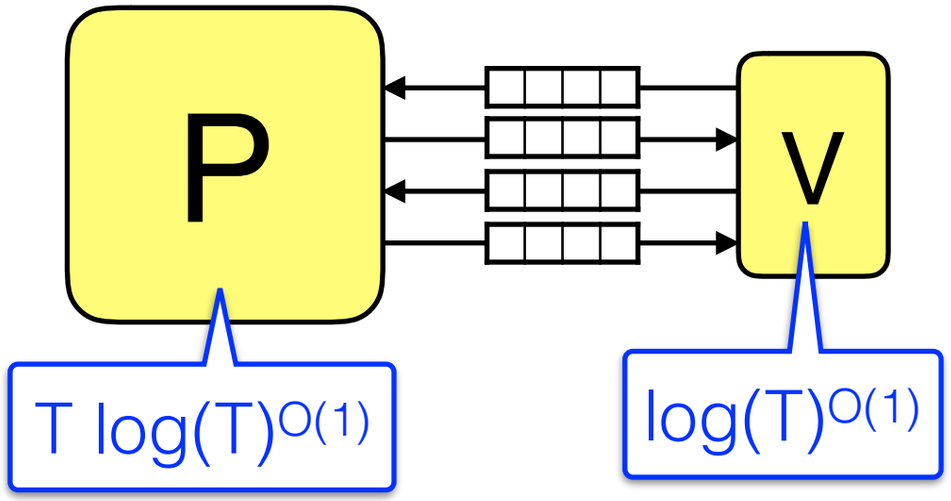

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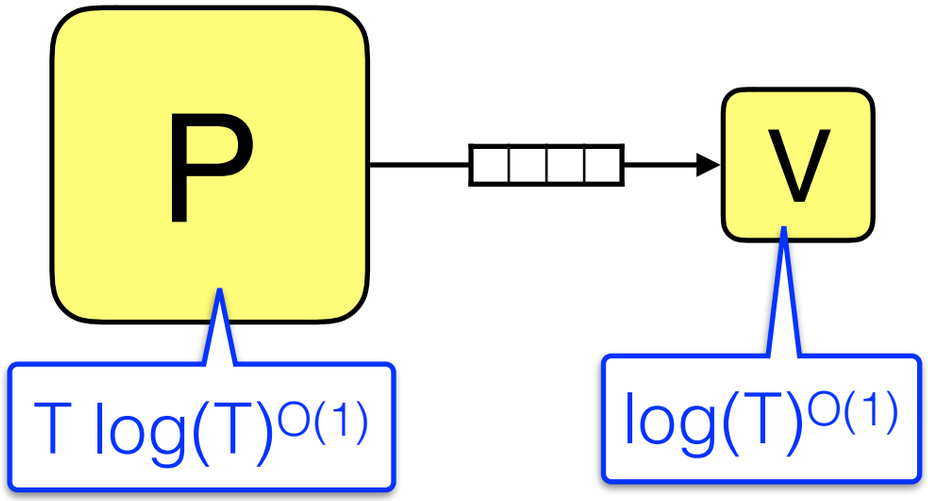

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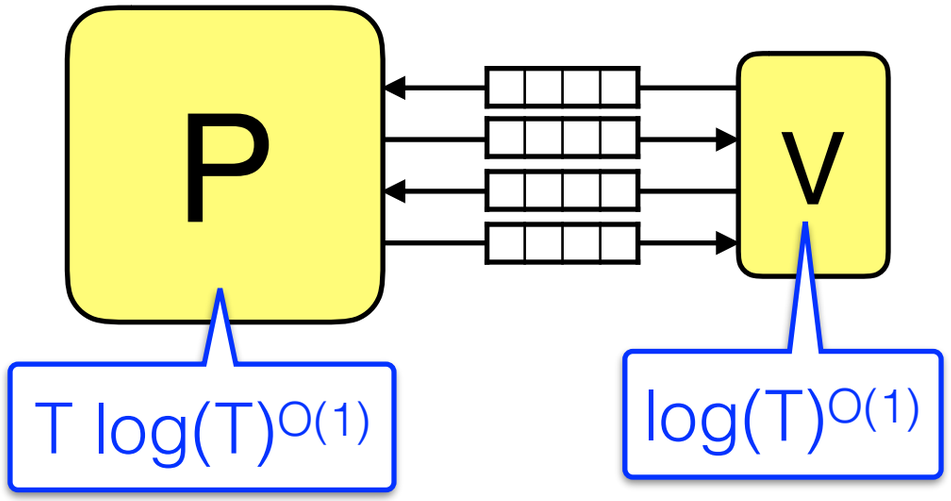

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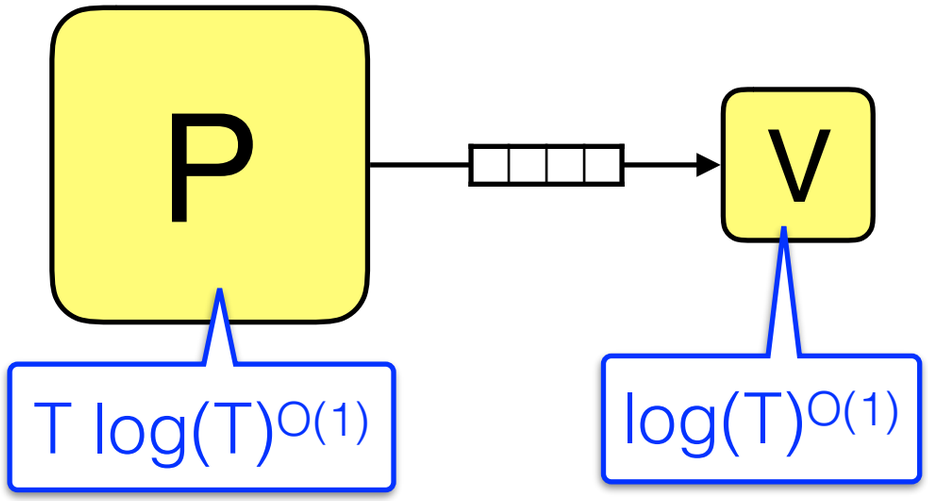

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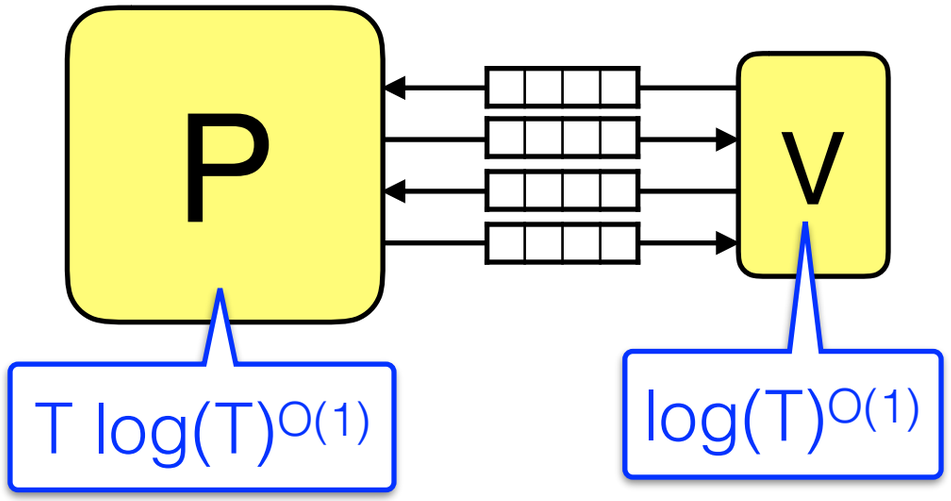

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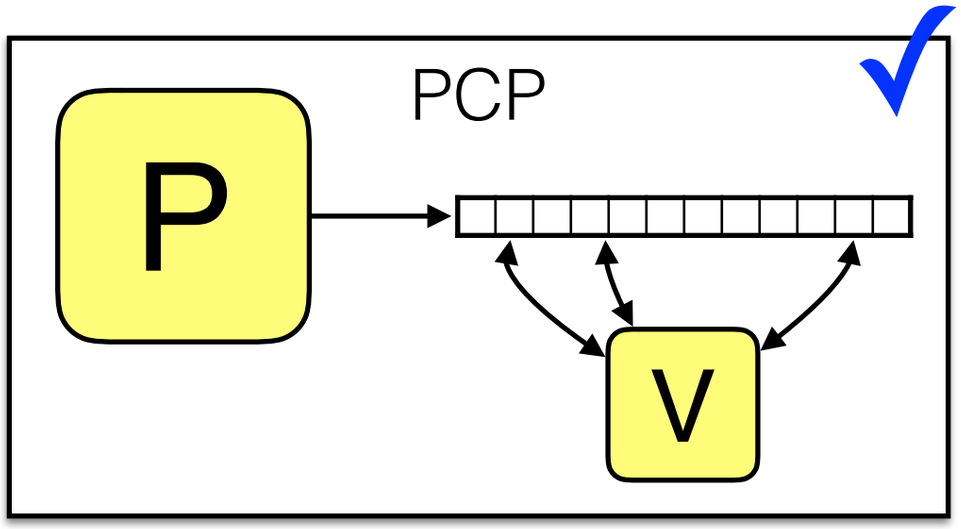

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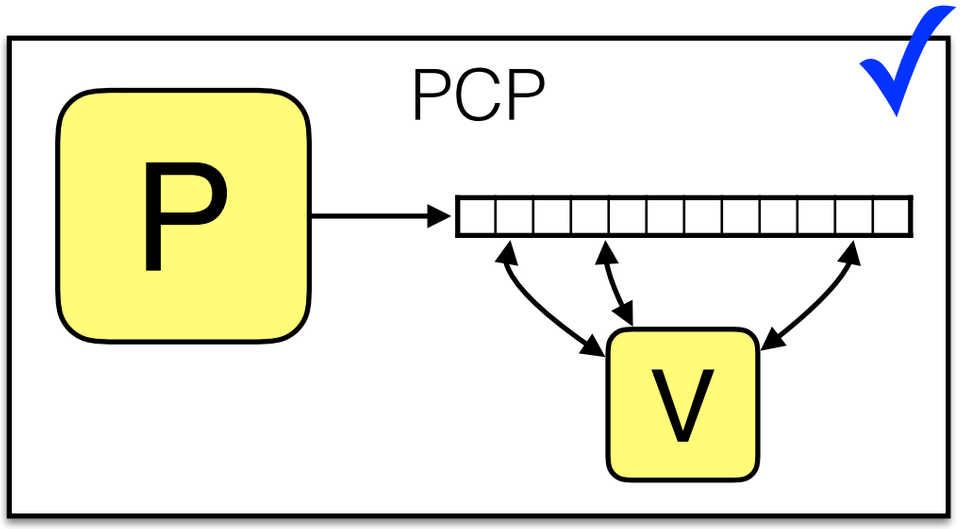

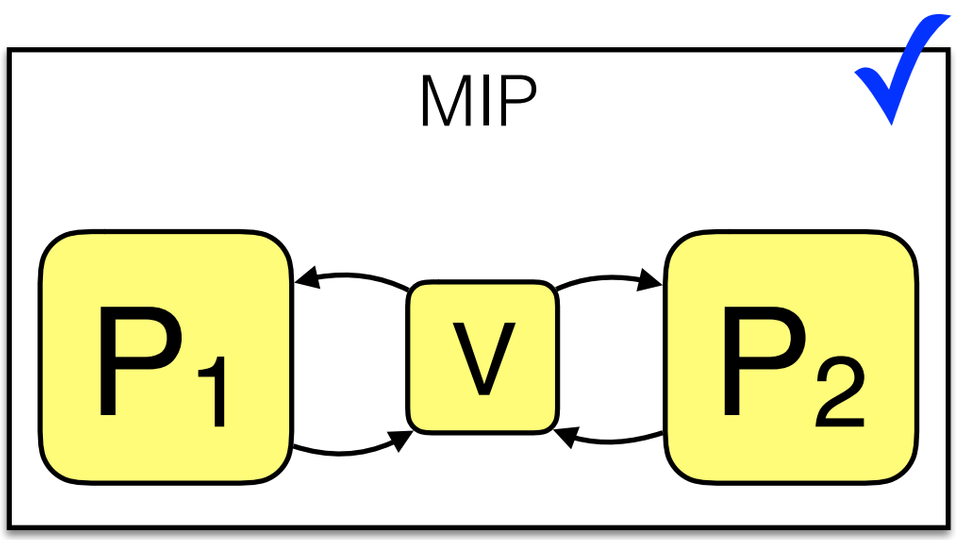

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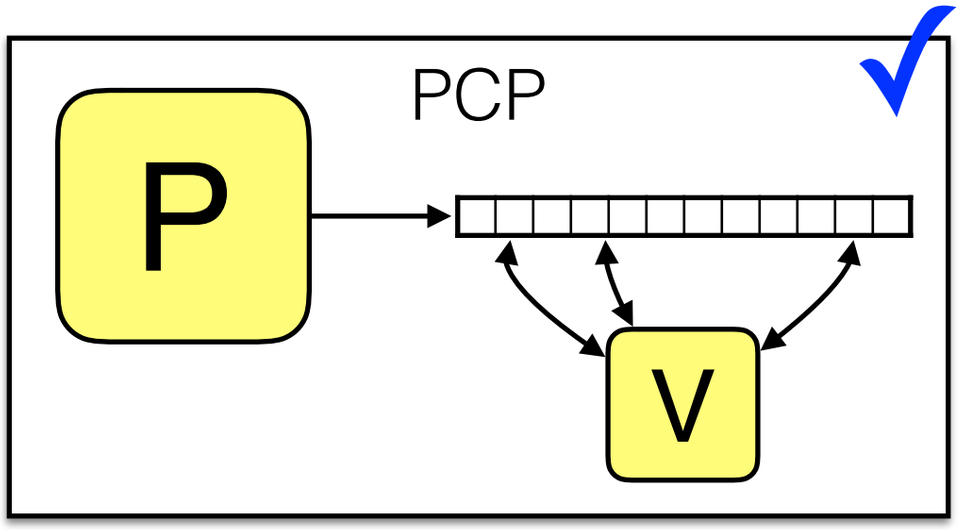

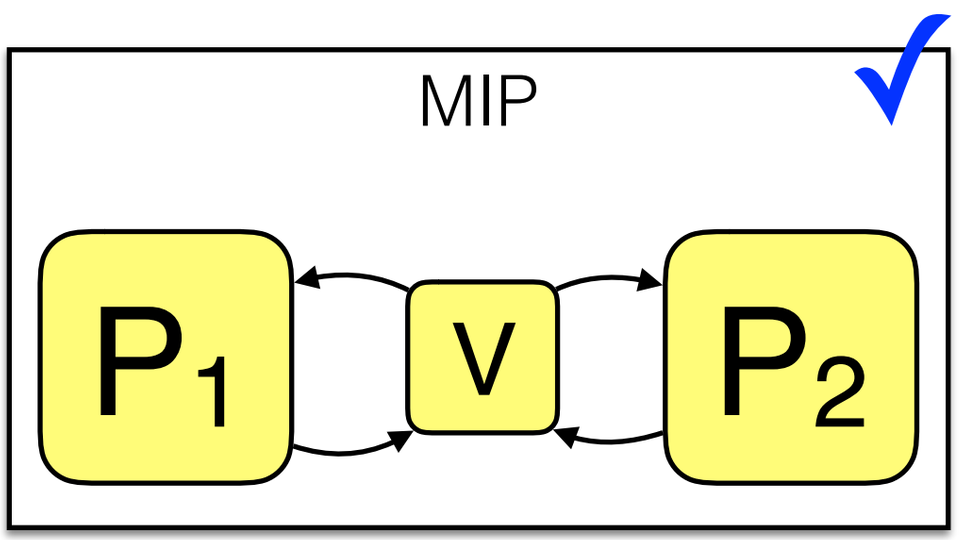

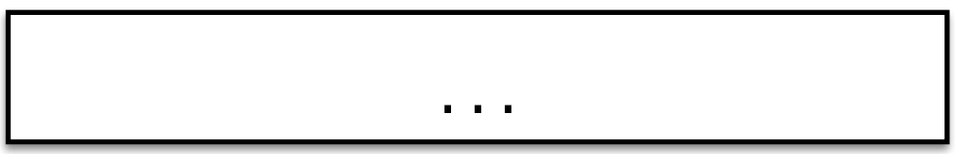

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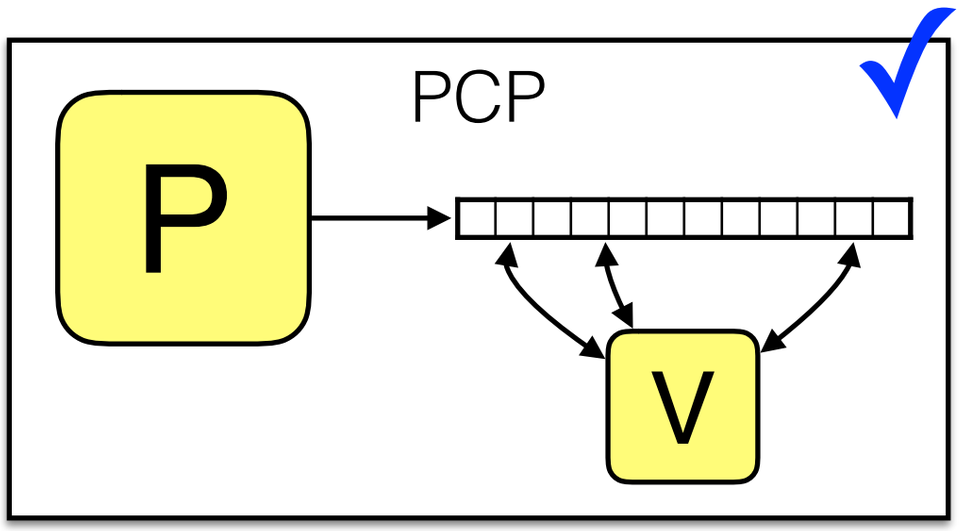
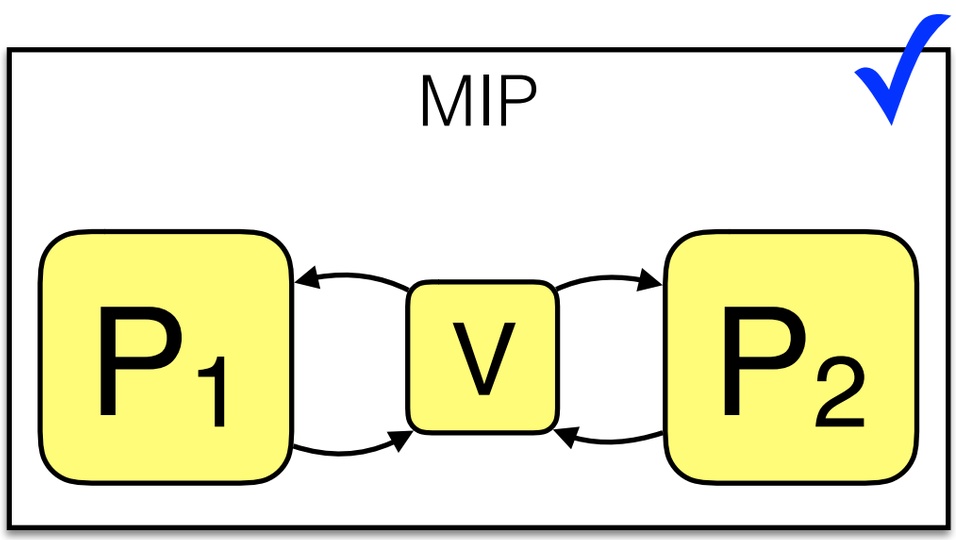
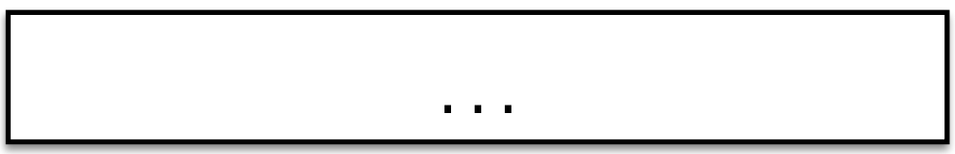


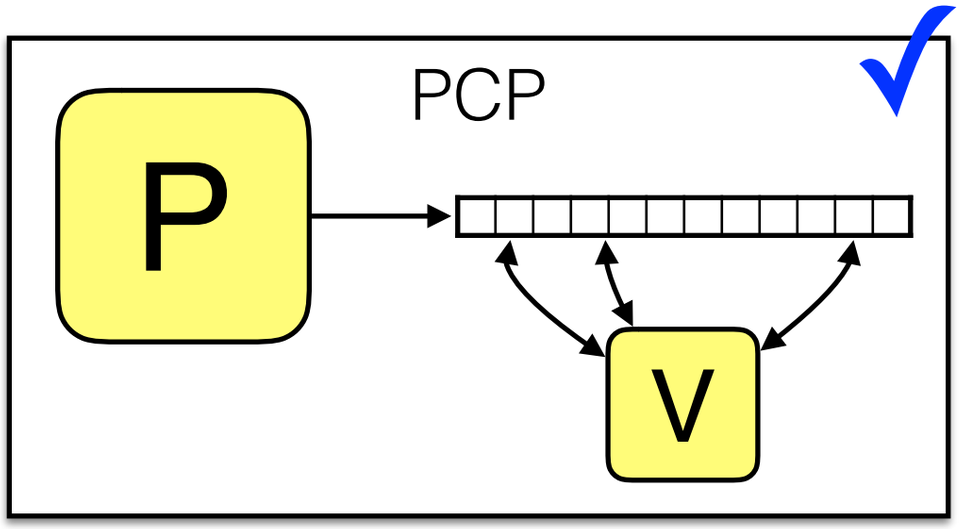


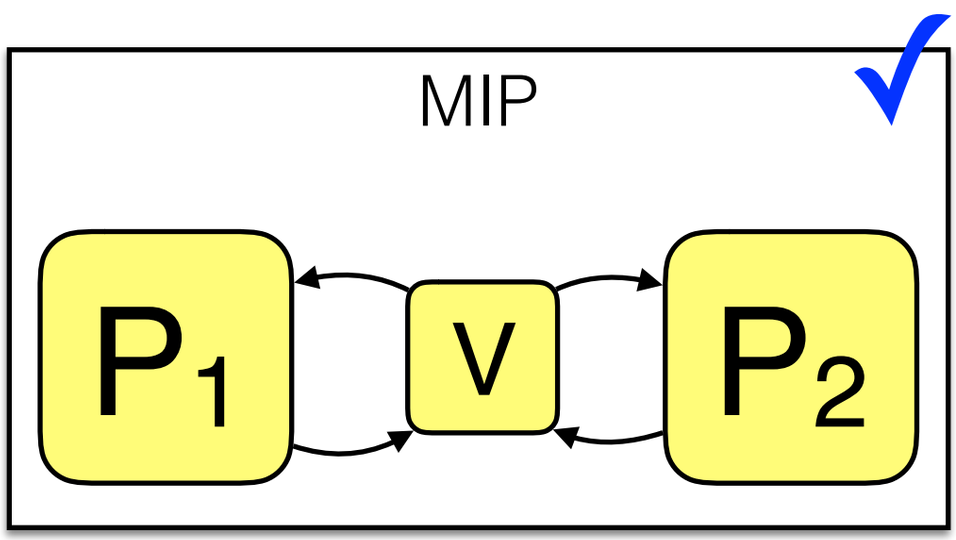


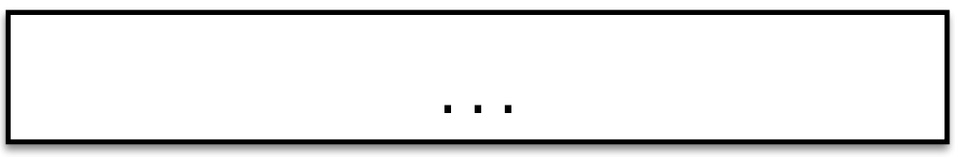
Convenient Succinct Proofs

An Awkward Situation

Inconvenient Succinct Proofs







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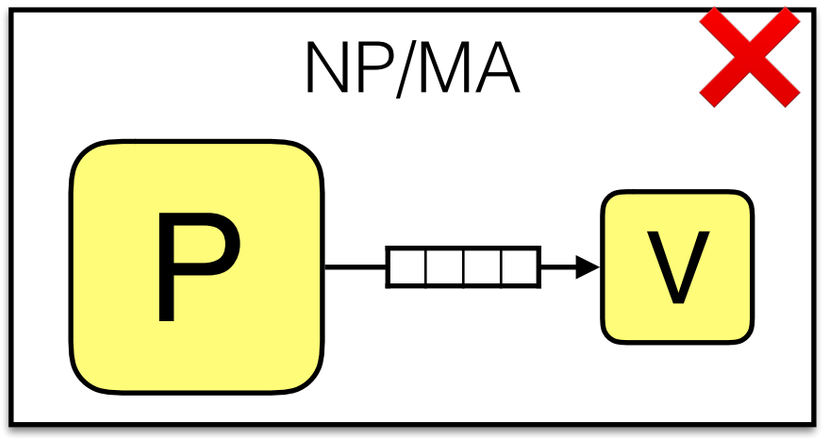
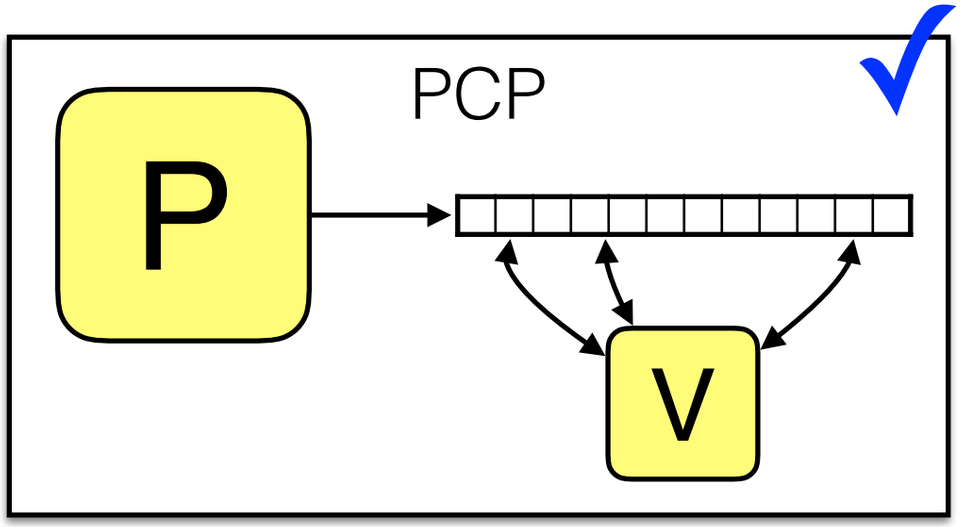
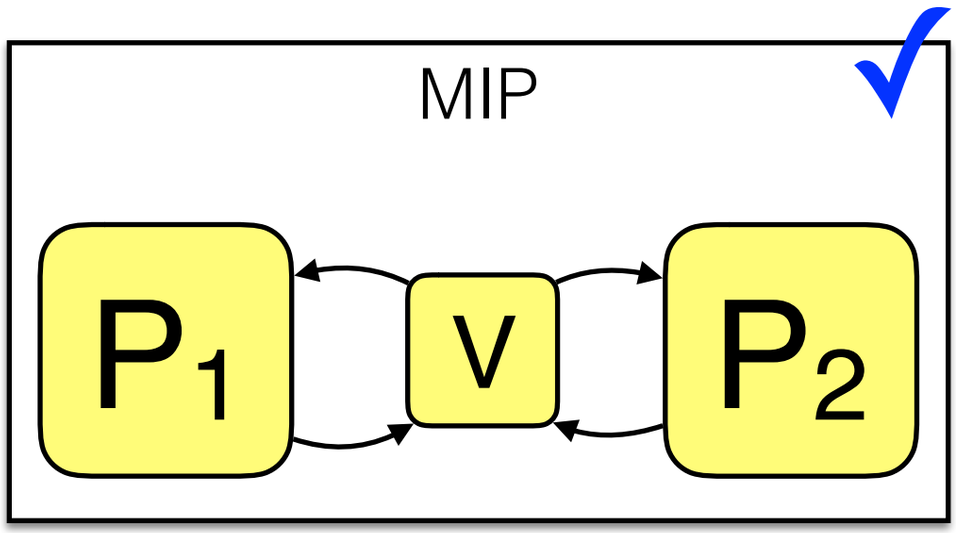
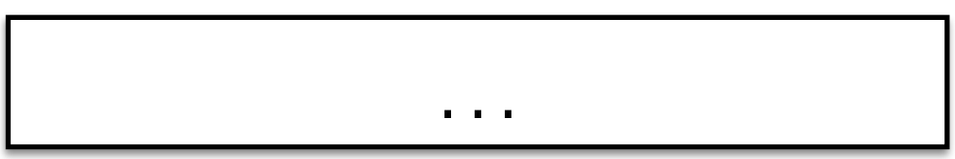


An Awkward Situation

Inconvenient Succinct Proofs







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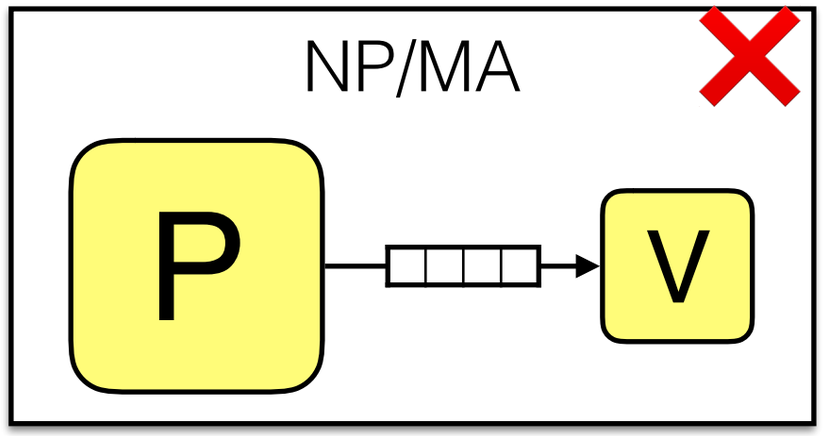
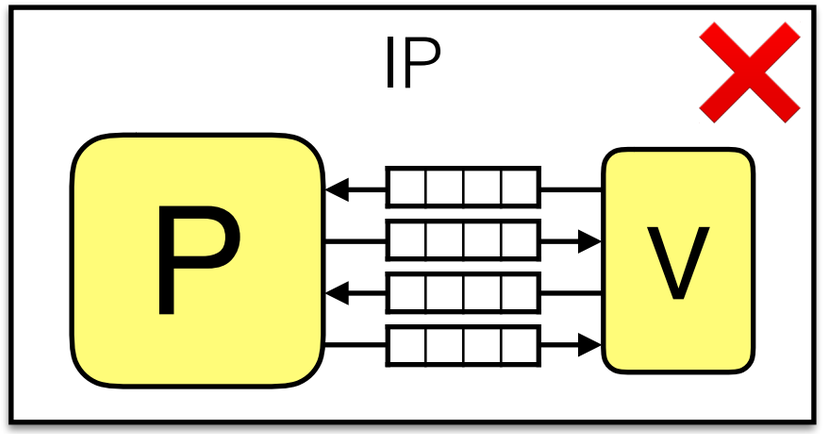
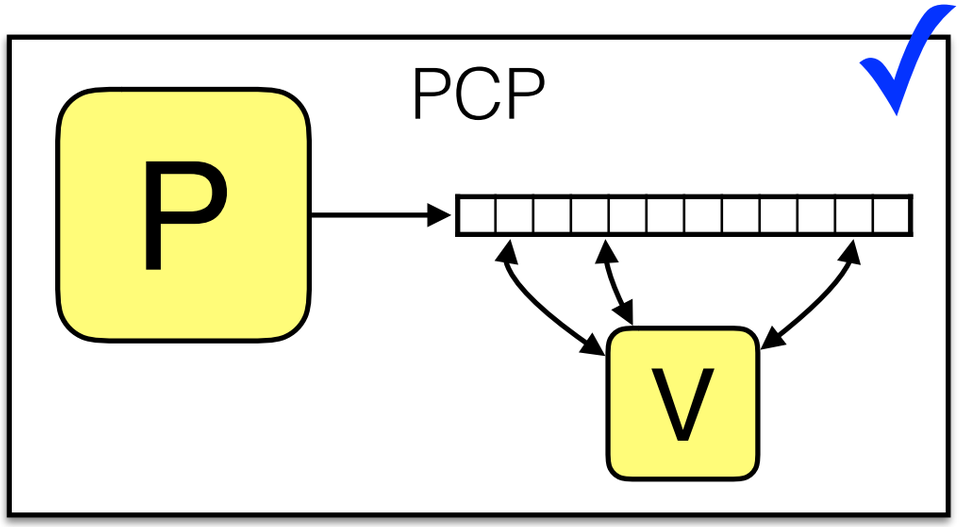
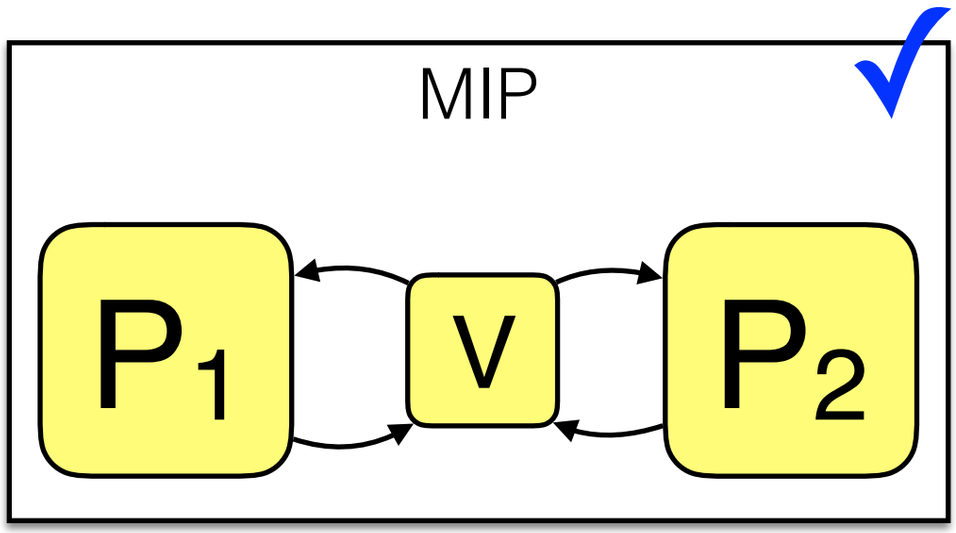




Cryptography As A Bridge

Inconvenient Succinct Proofs





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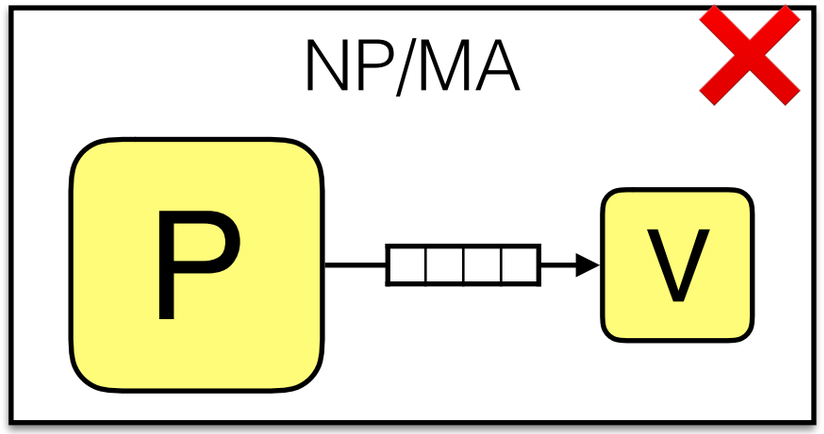
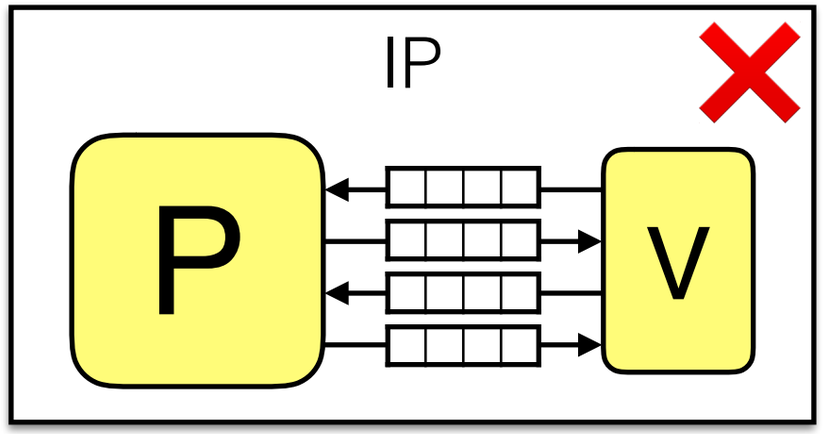
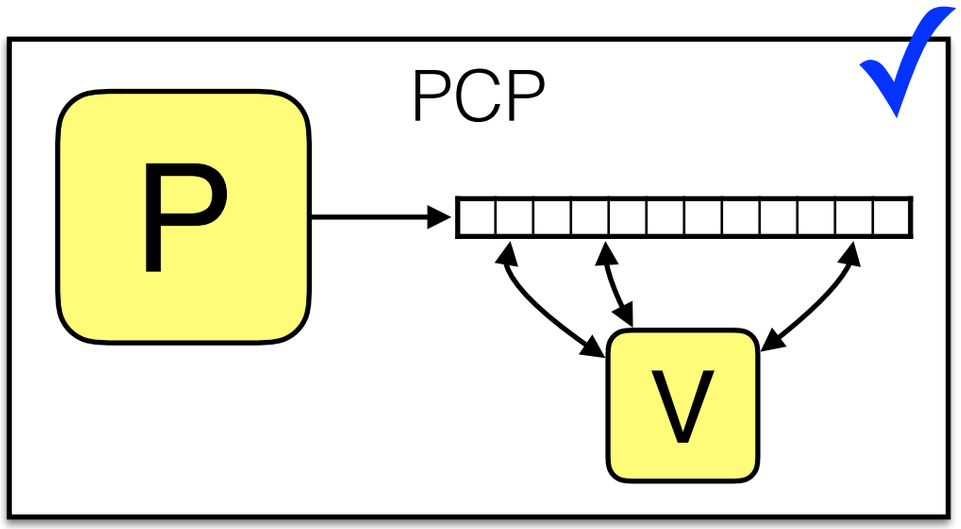
Convenient Succinct Proofs

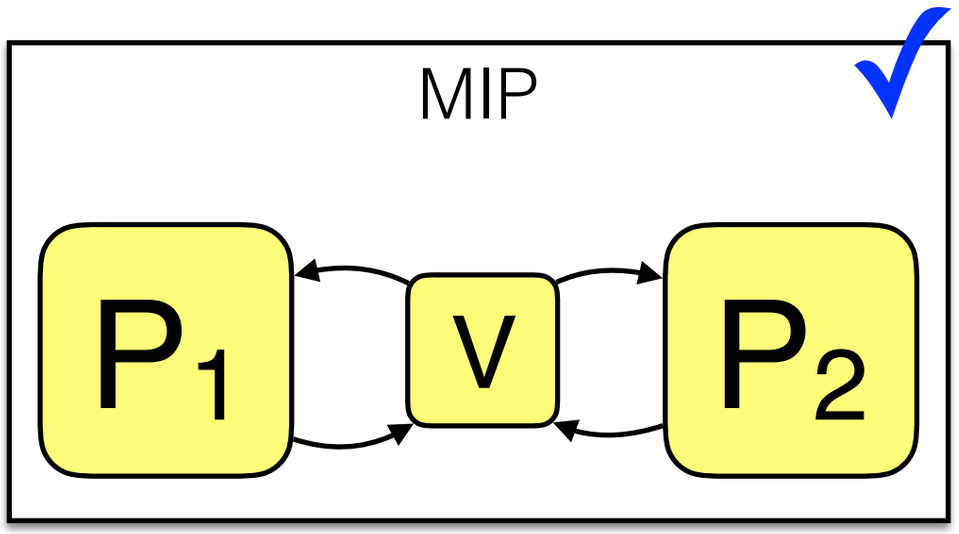


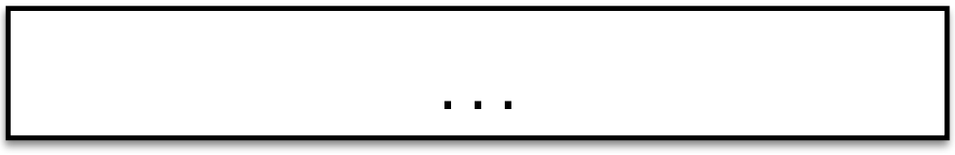


Cryptography As A Bridge

Inconvenient Succinct Proofs

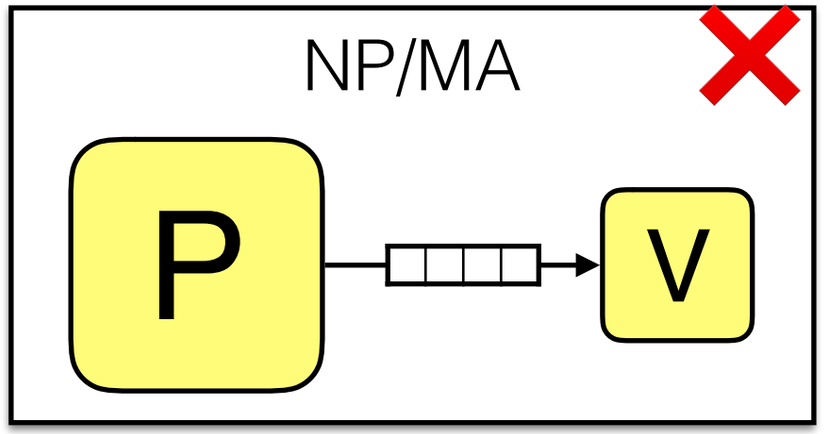


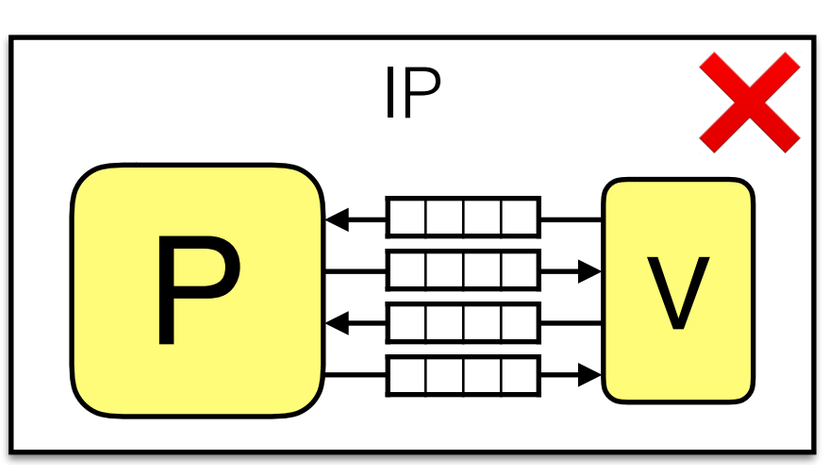




CRYPTO

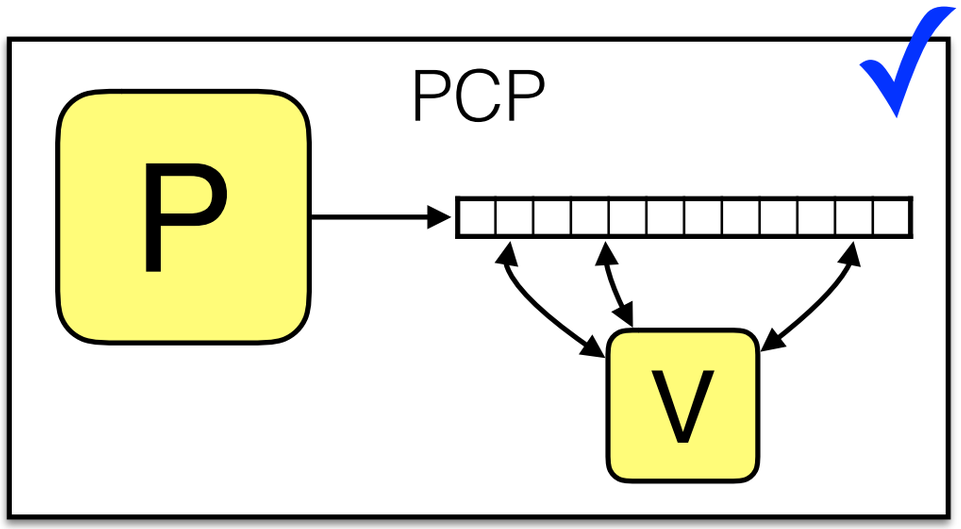
Convenient Succinct Proofs

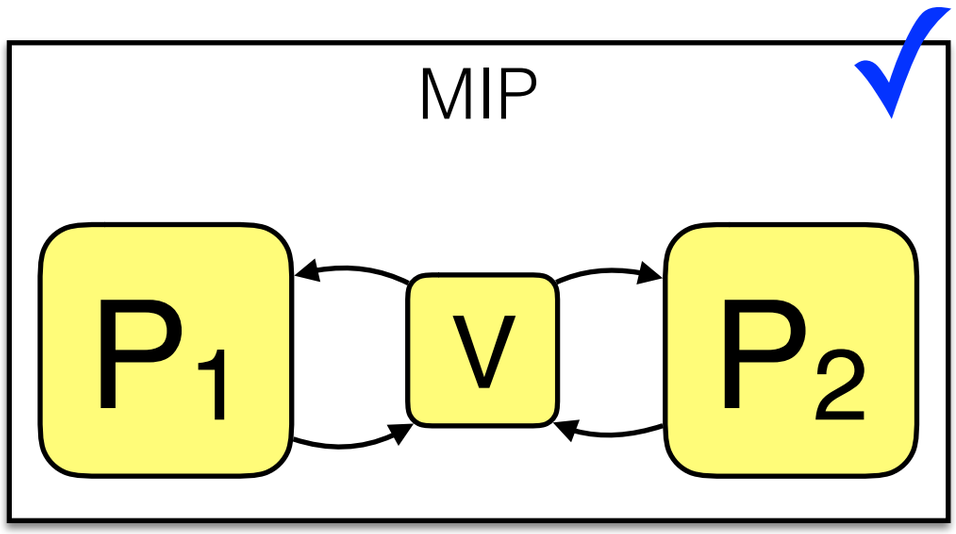


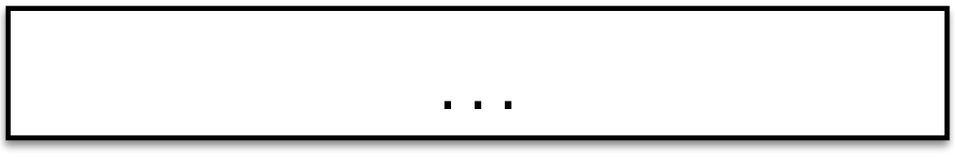


Cryptography As A Bridge

Inconvenient Succinct Proofs



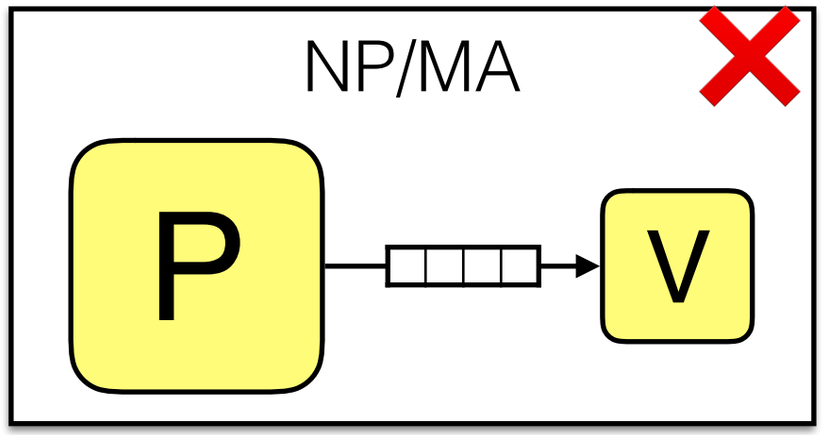


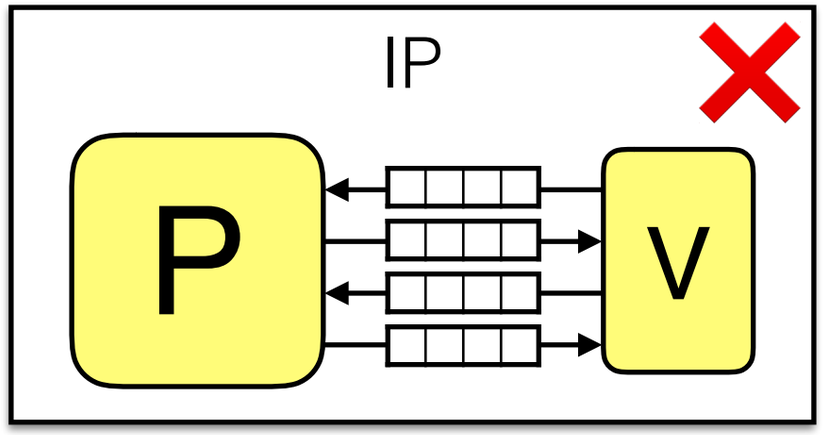


CRYPTO

Arguments

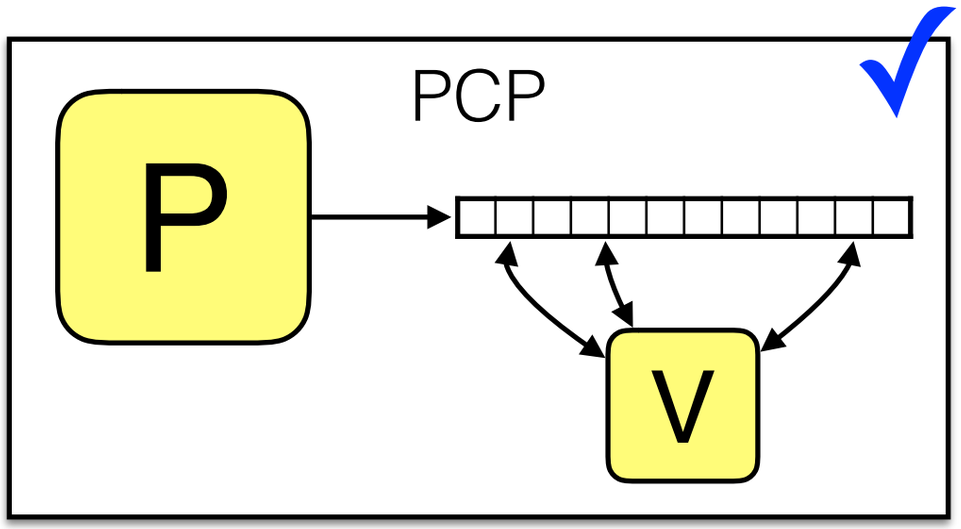
Convenient Succinct ~~Proofs~~

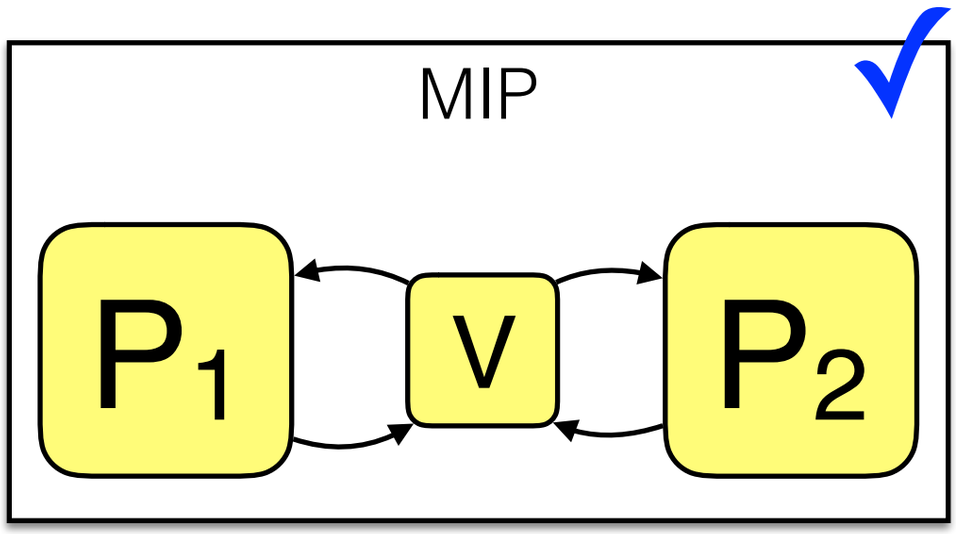


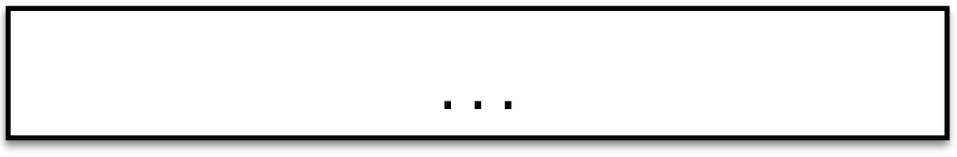


Cryptography As A Bridge

Inconvenient Succinct Proofs



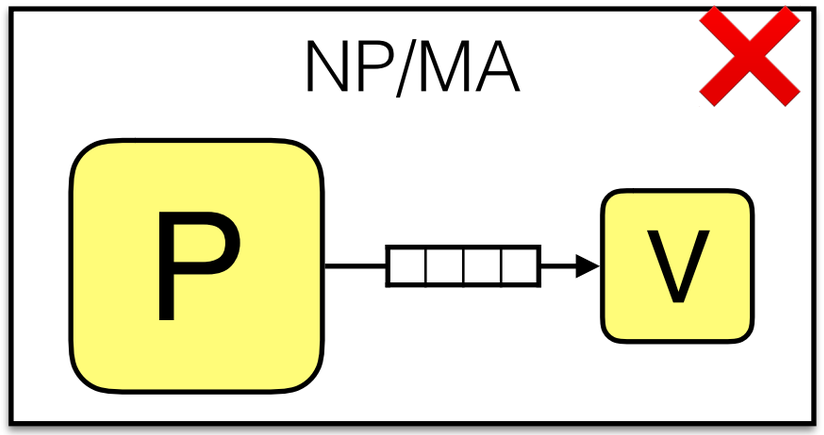


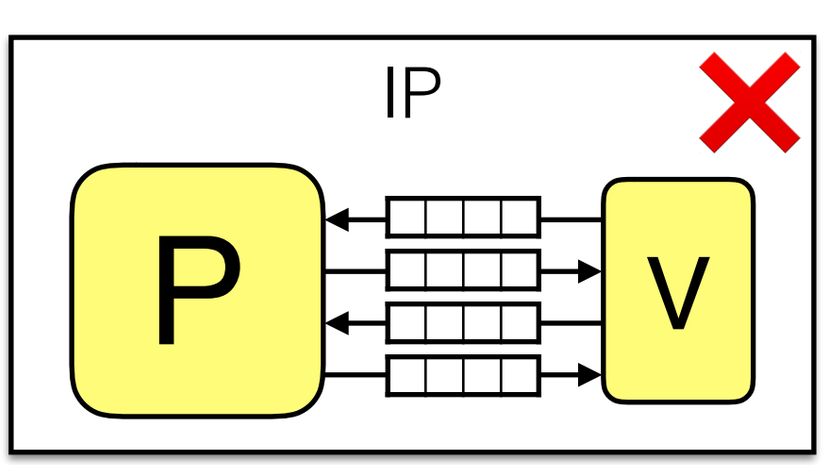


CRYPTO

Arguments

Convenient Succinct ~~Proofs~~



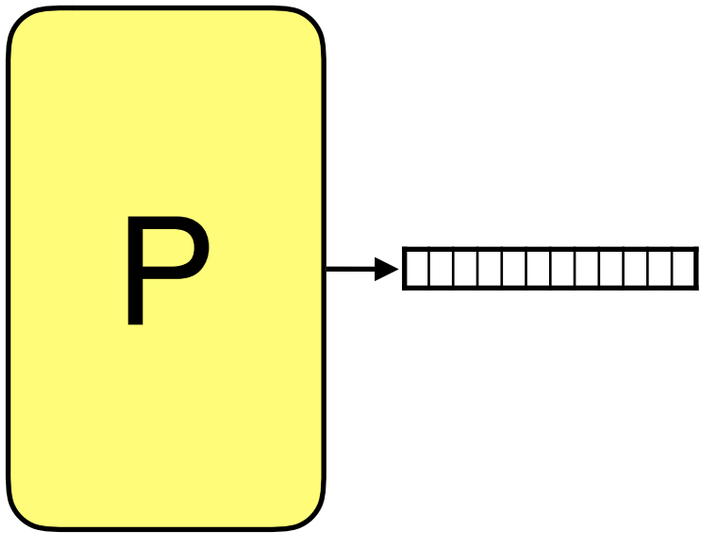


A very productive approach
to construct *succinct arguments*

Constructions

From **PCPs**

From **PCPs**



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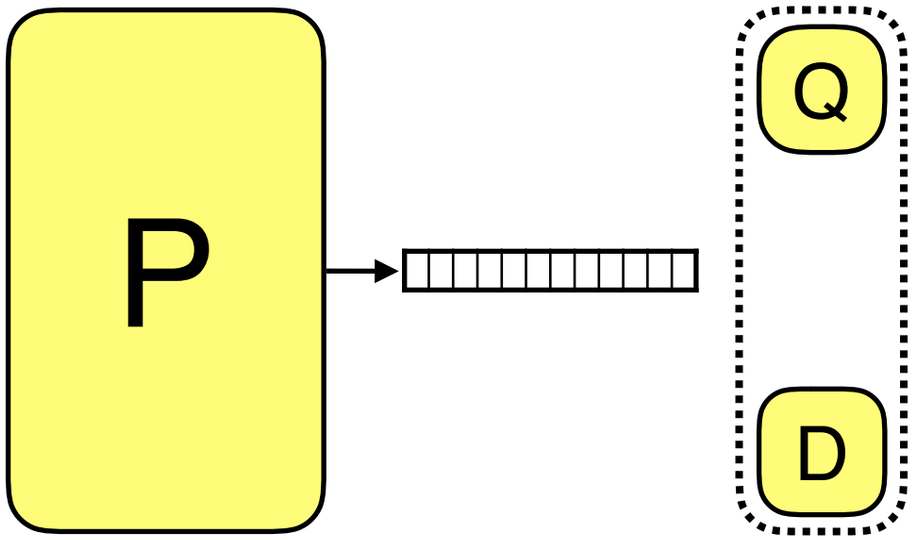

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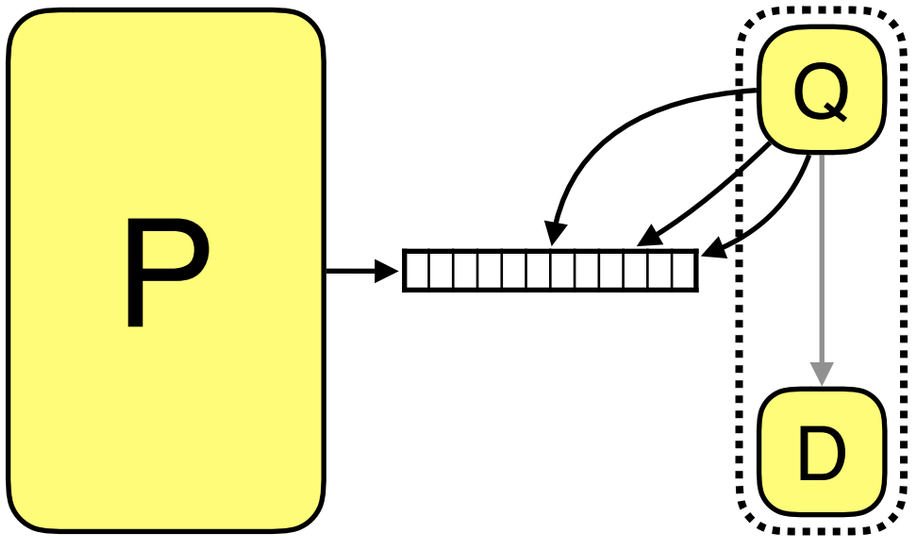

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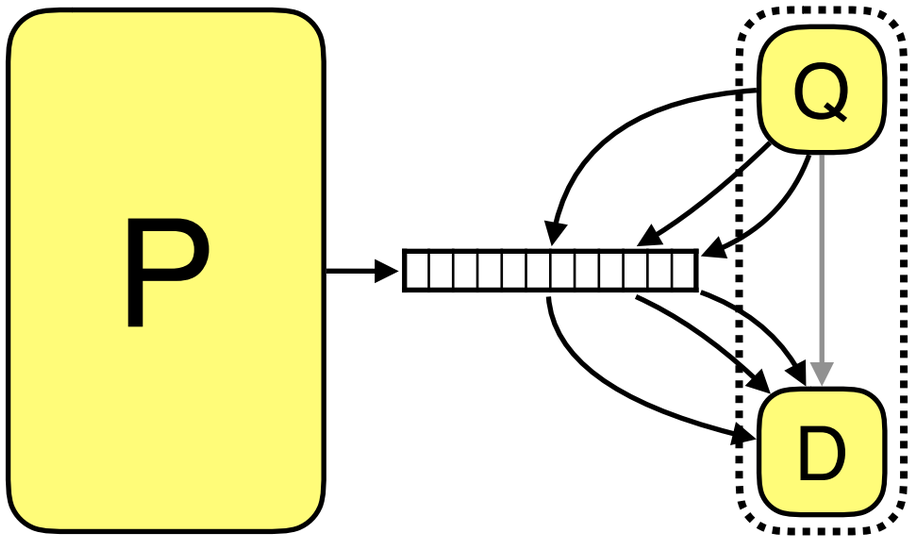

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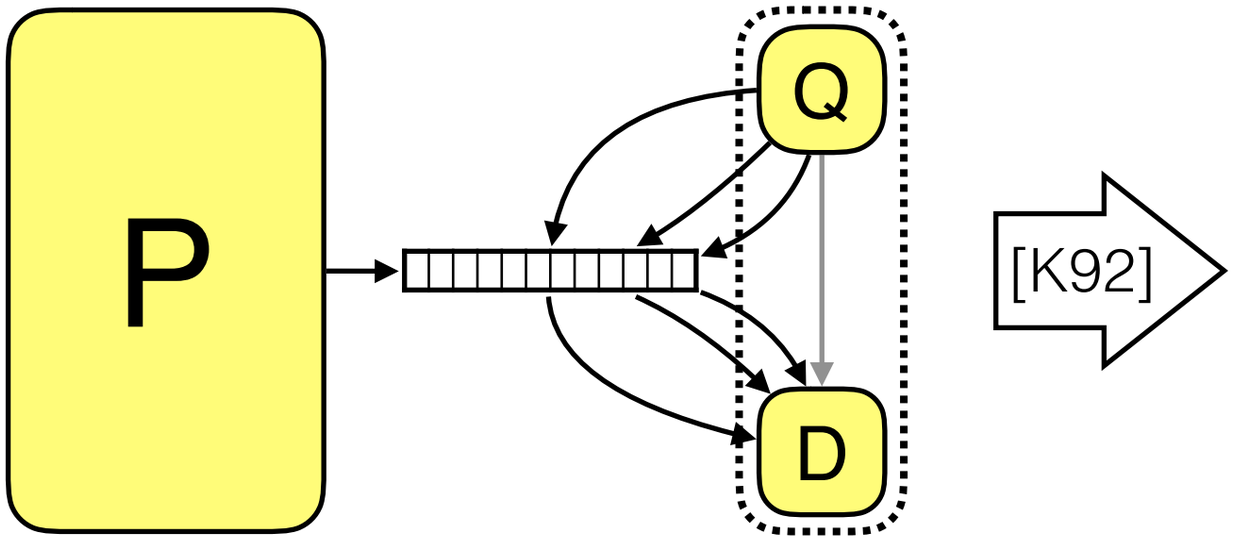

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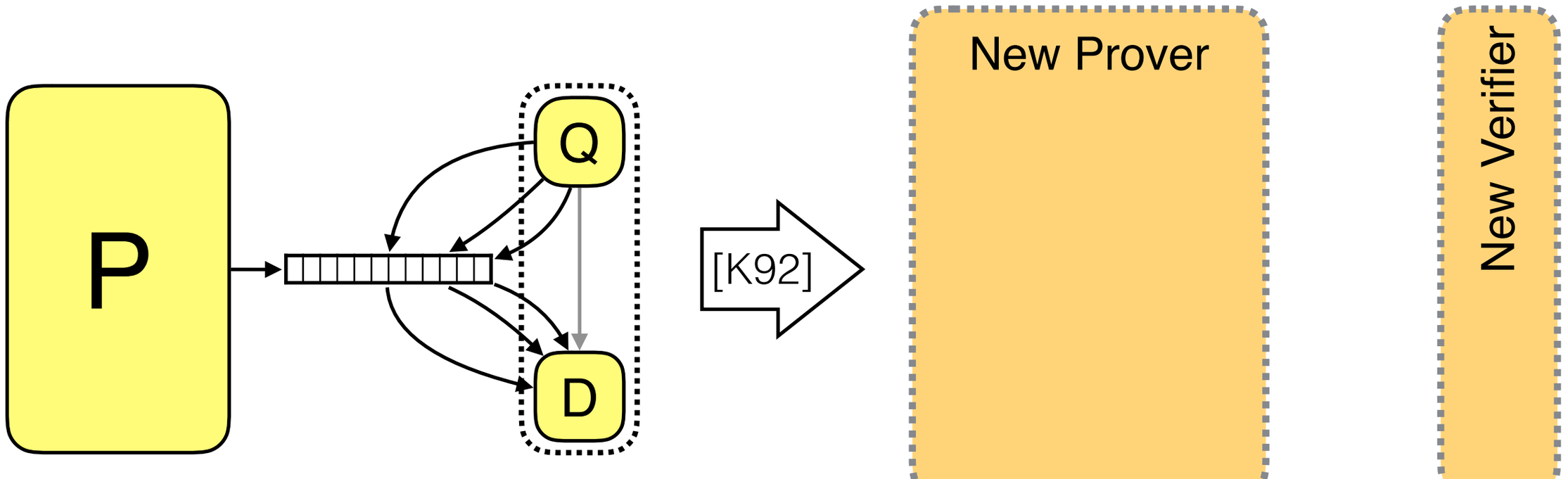

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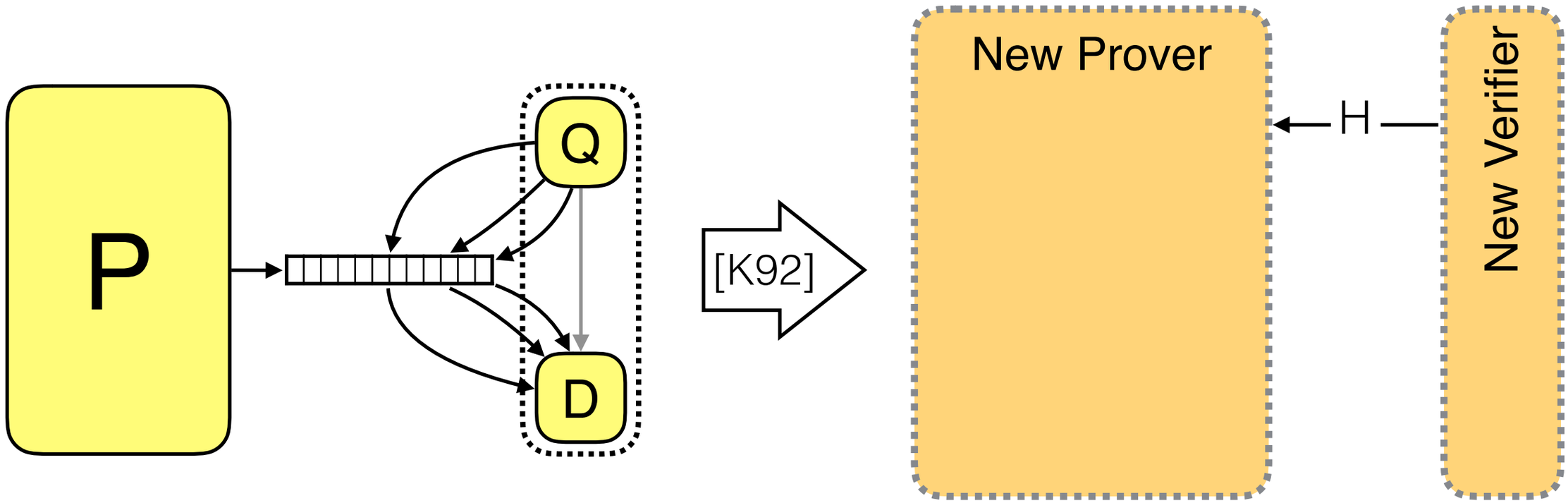

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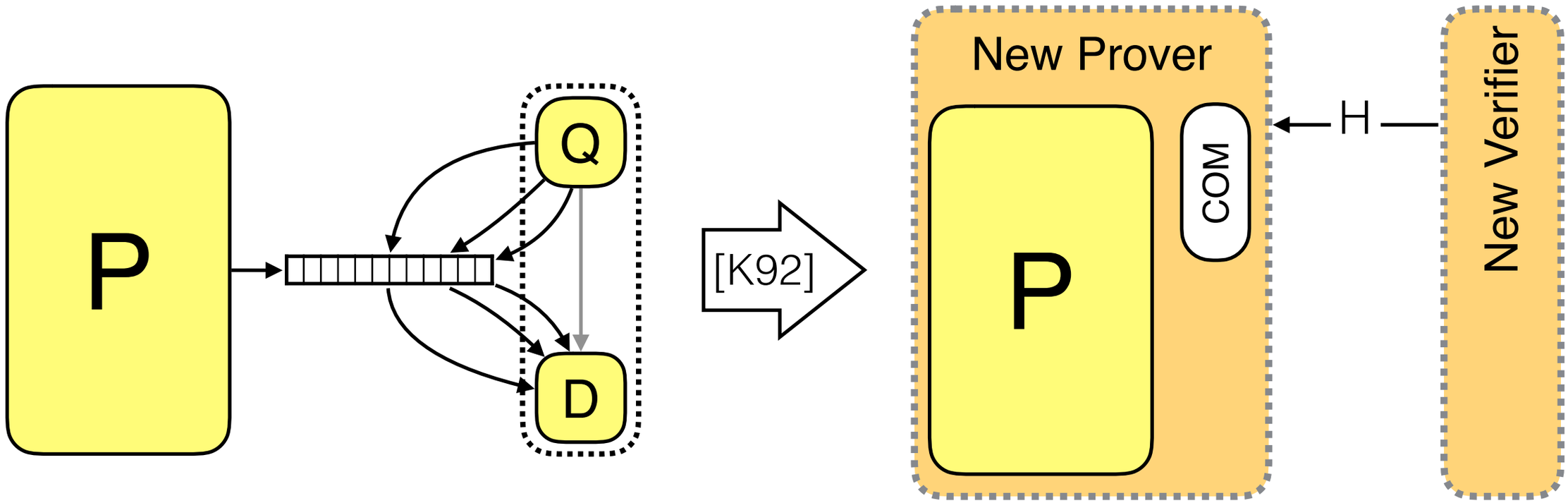

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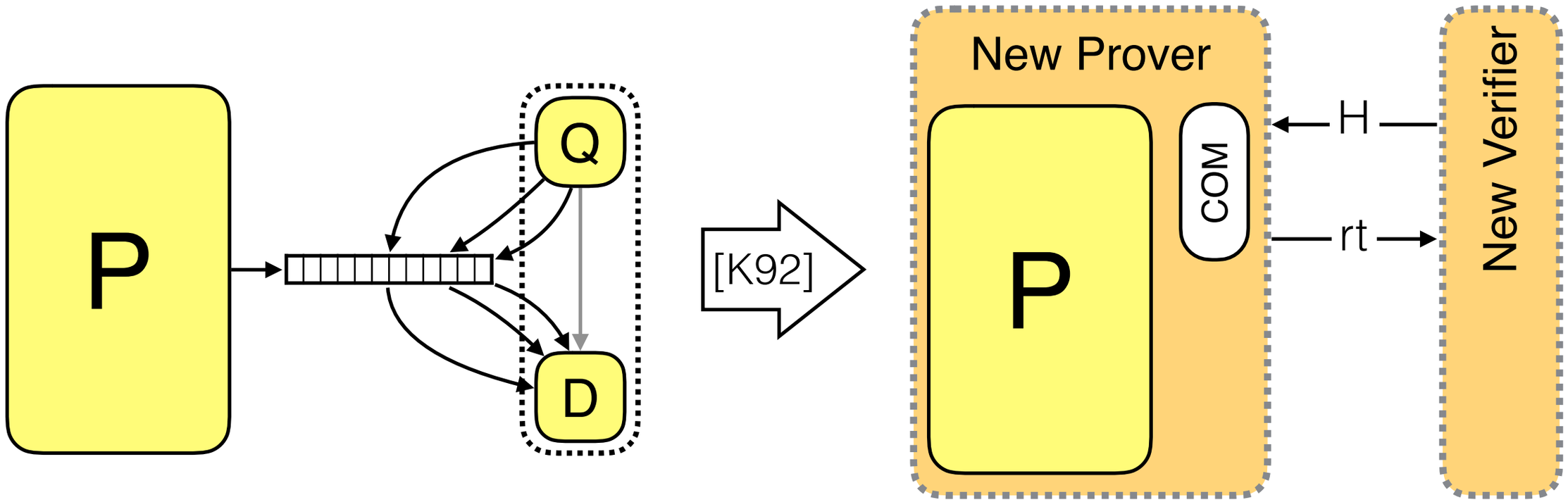

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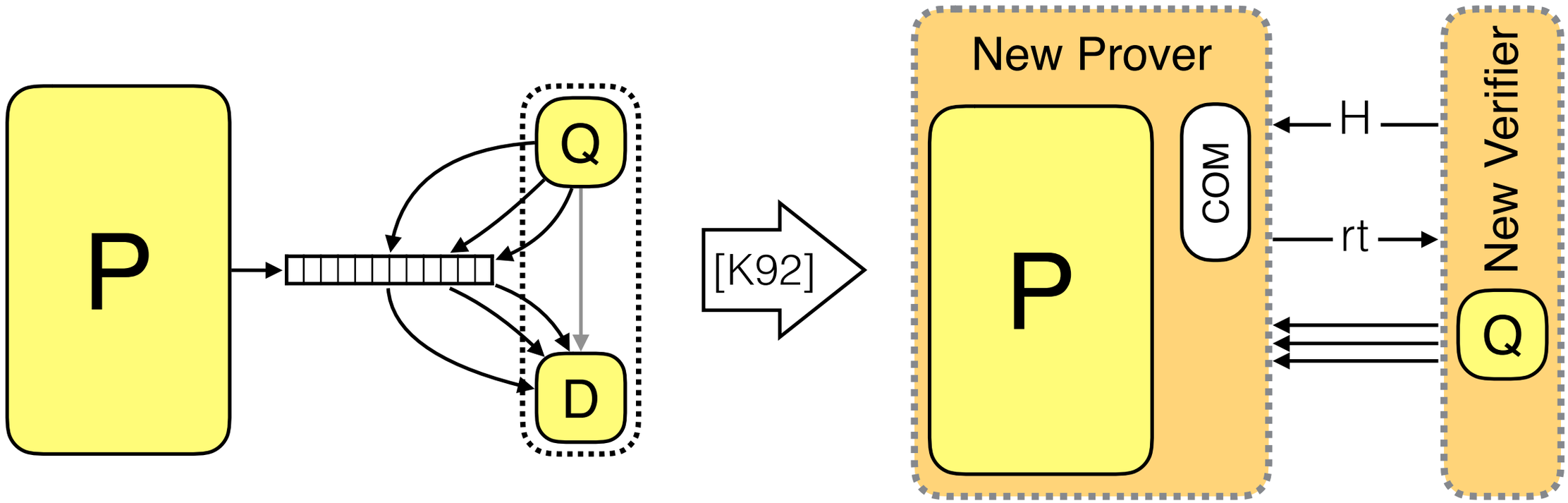

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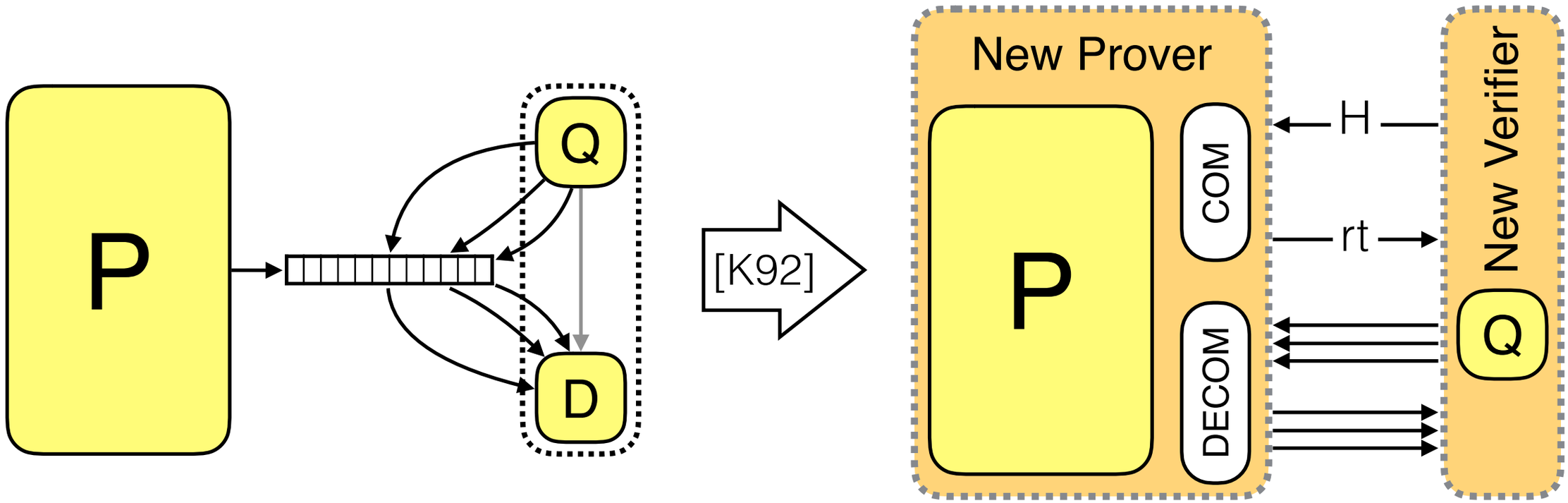

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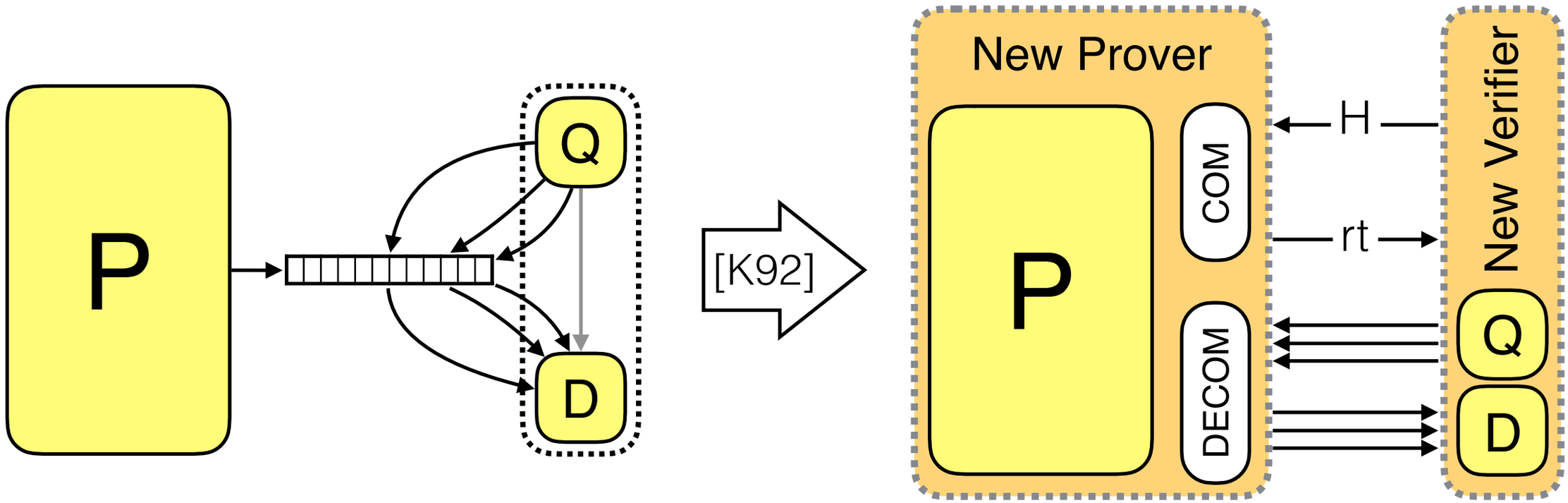

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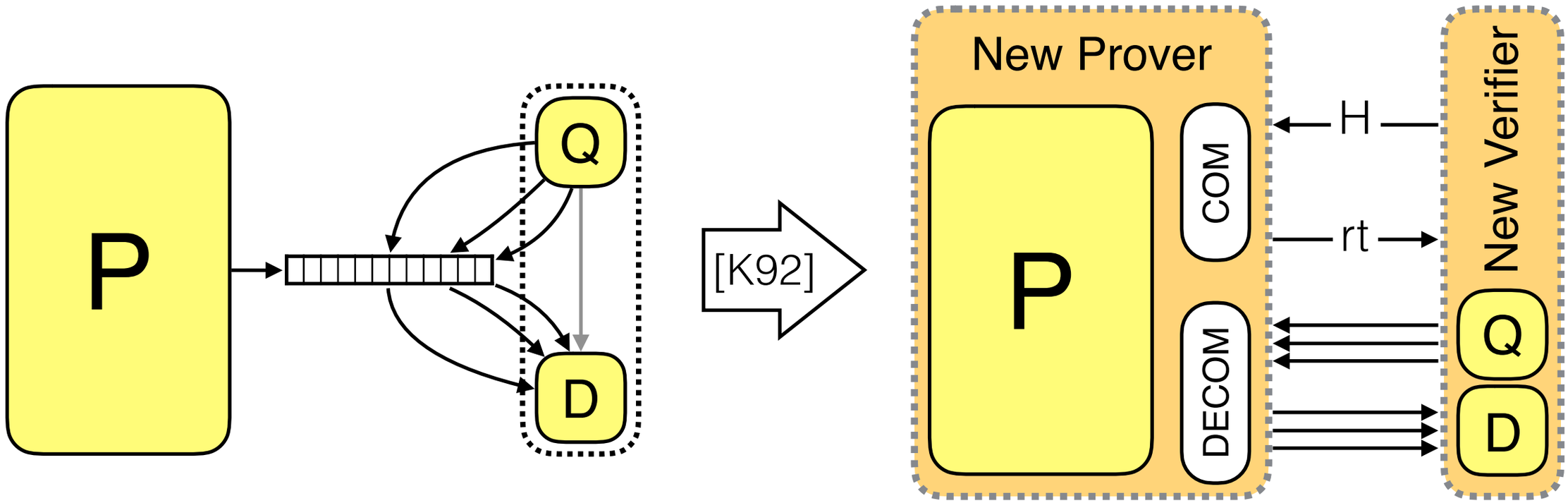

work	number of messages	cryptographic tool
[K92][BG08]	4	collision-resistant hashing
[BKP17]	3	multicollision-resistant hashing
[BCCGLRT16] [DFH12]	2	extractable collision-resistant hashing
[M94]	1	random oracle

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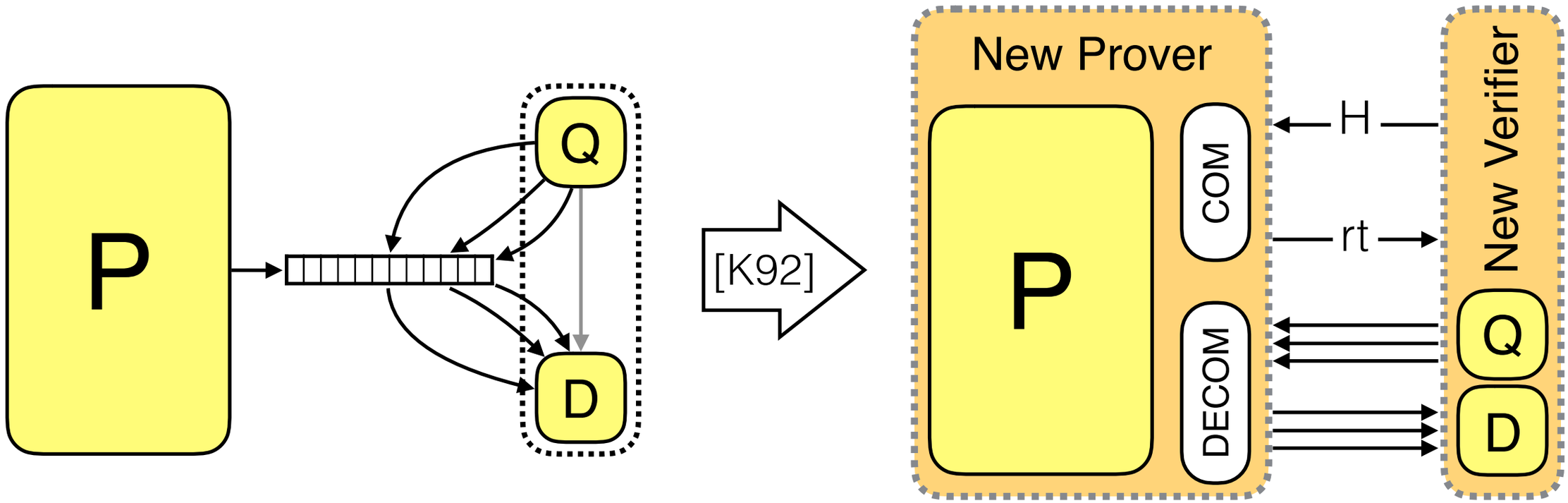

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From **MIPs**

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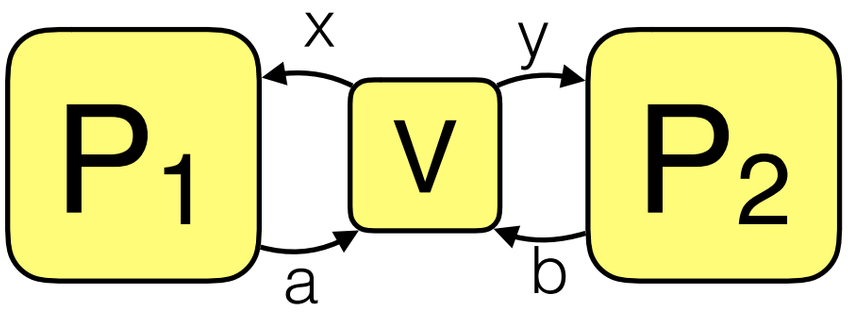

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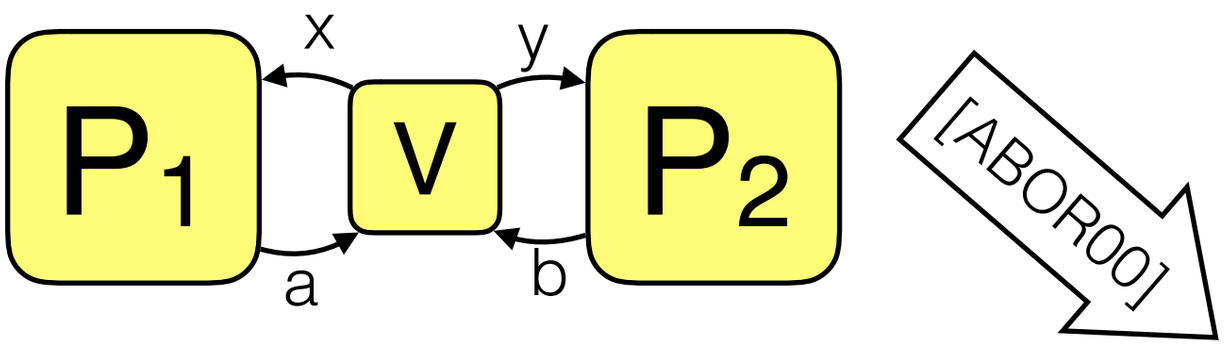

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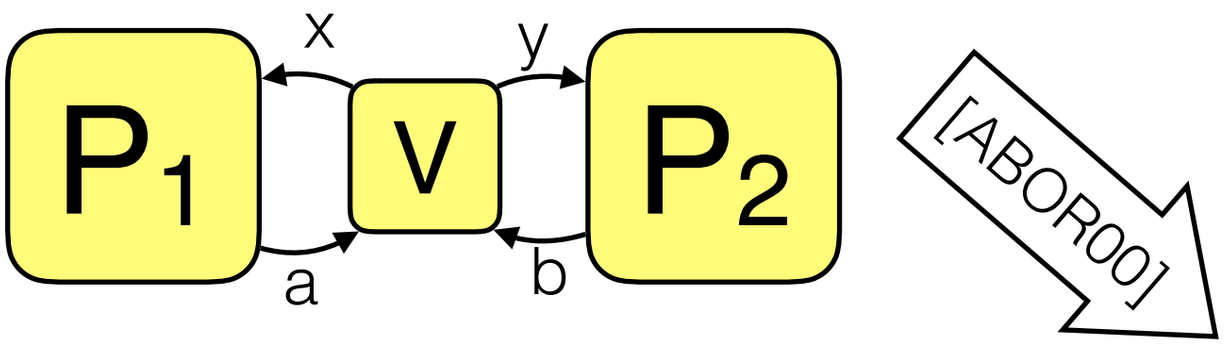

New Prover

New Verifier

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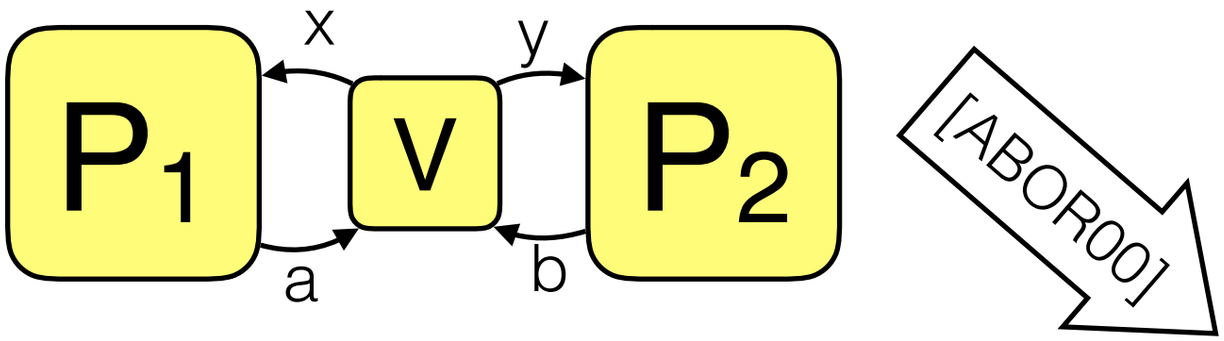

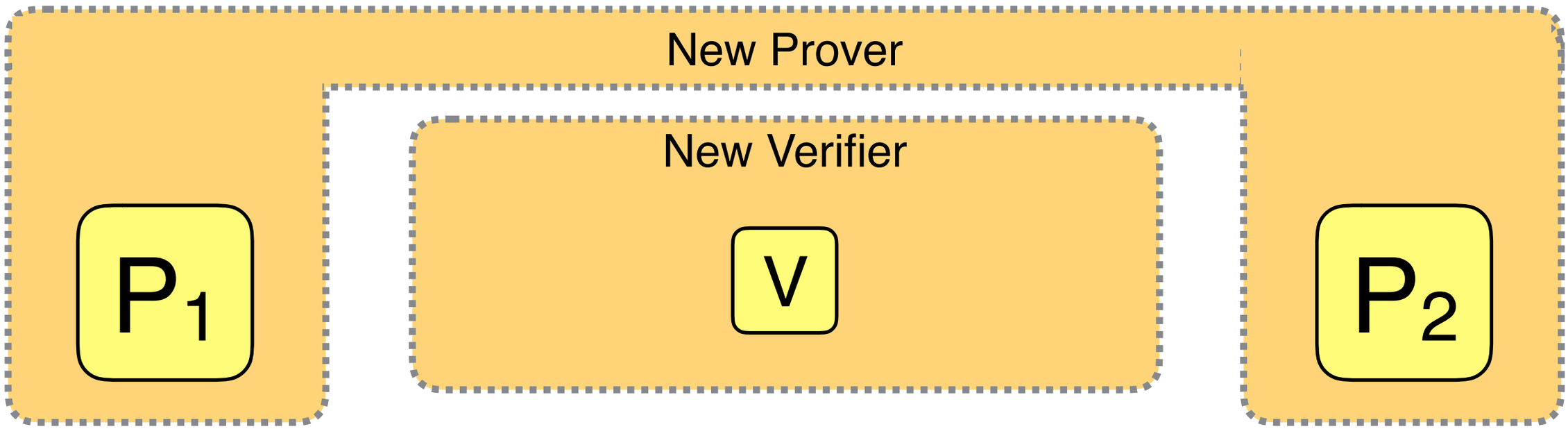

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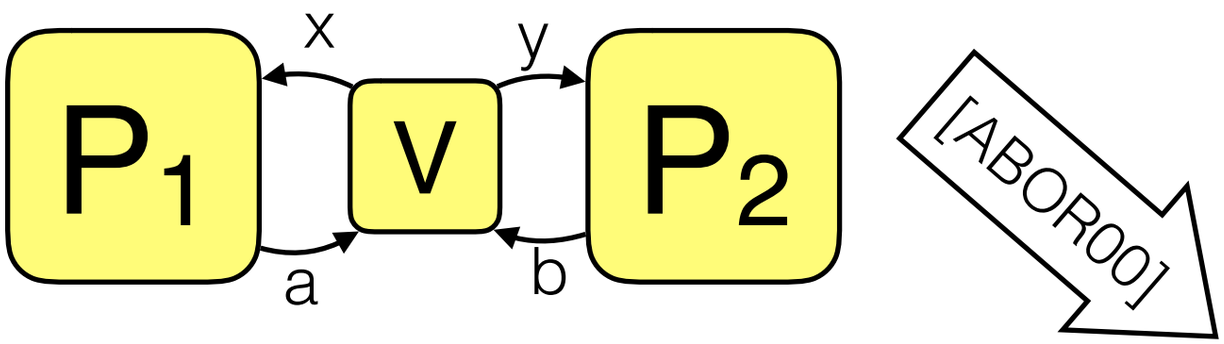

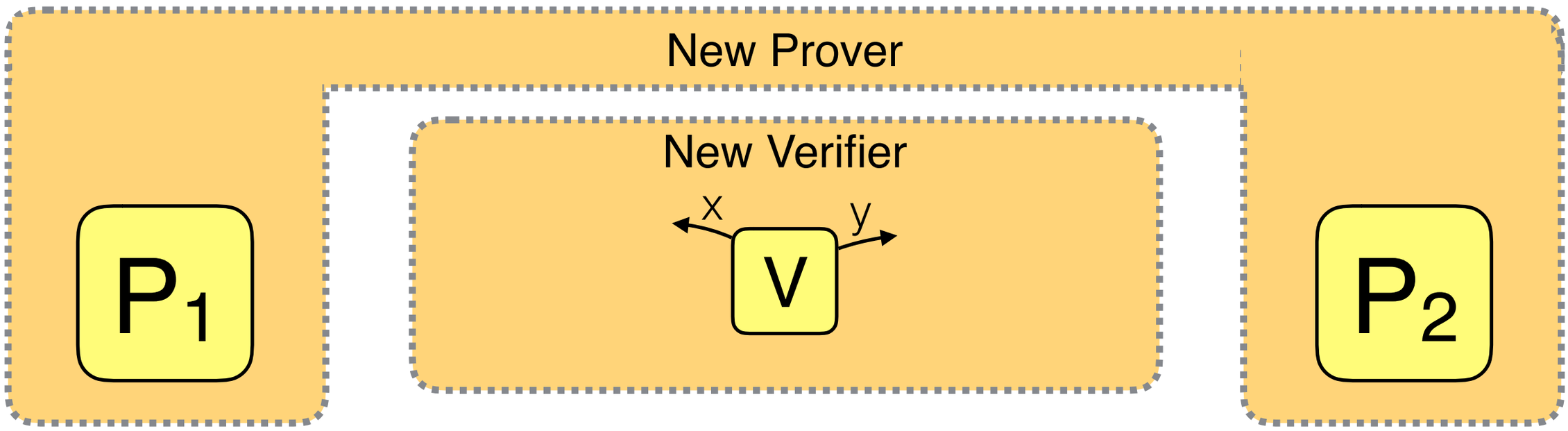

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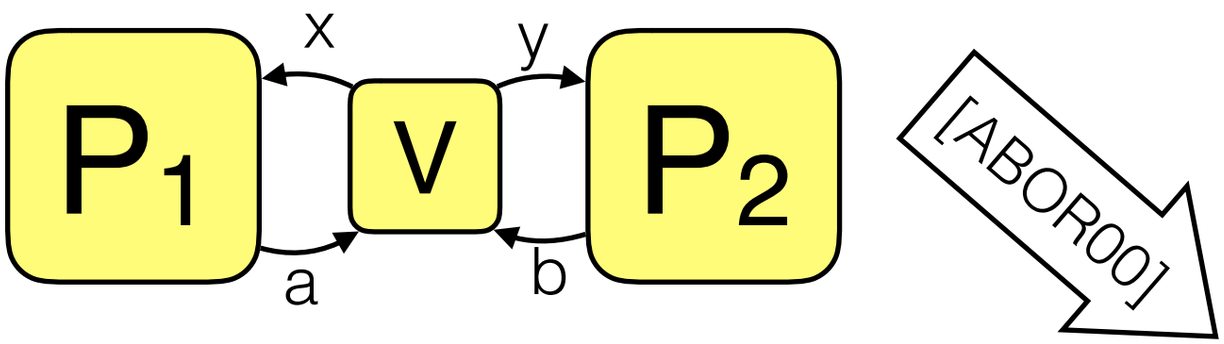
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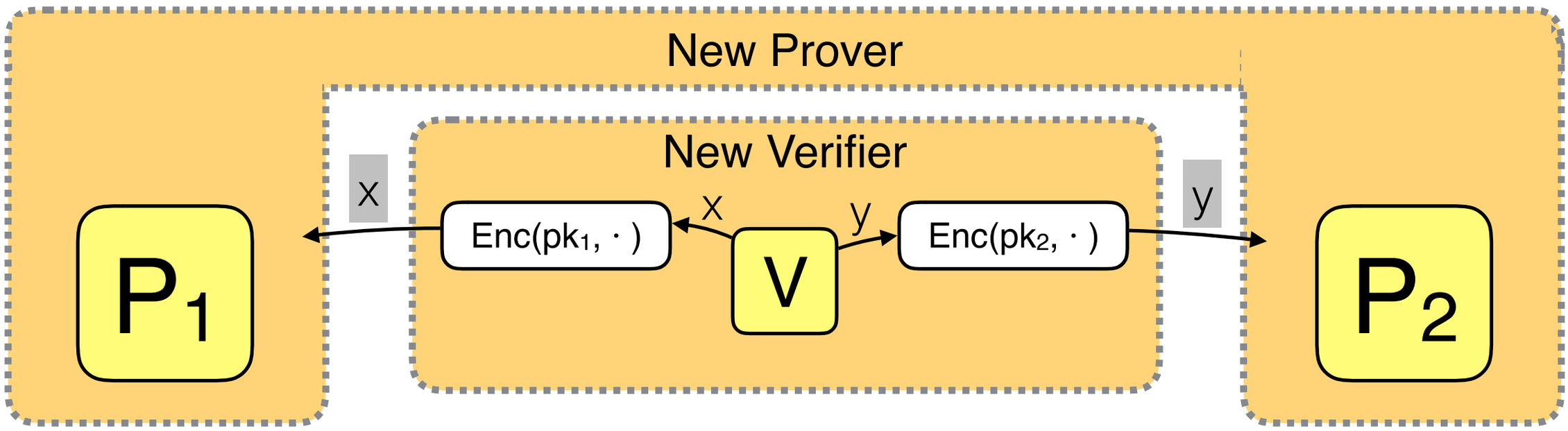
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New Verifier
x
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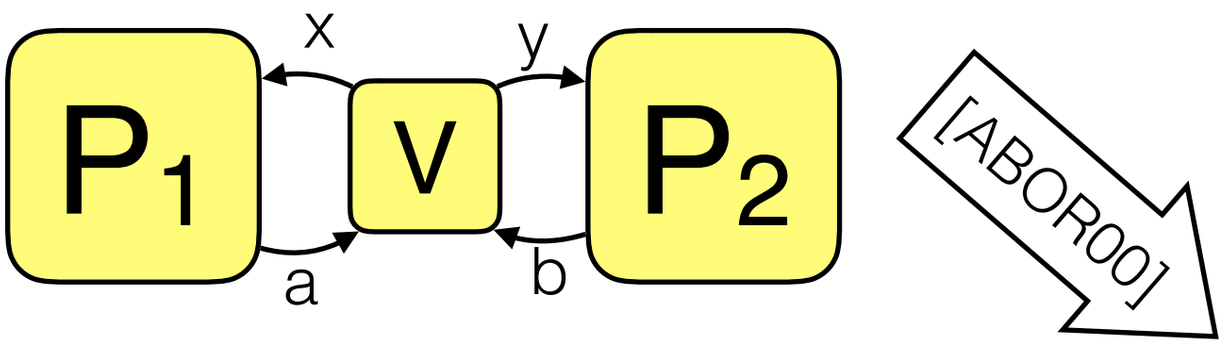
y
Enc(pk2, ·)
y
P1
V
P2

From **MIPs**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P2
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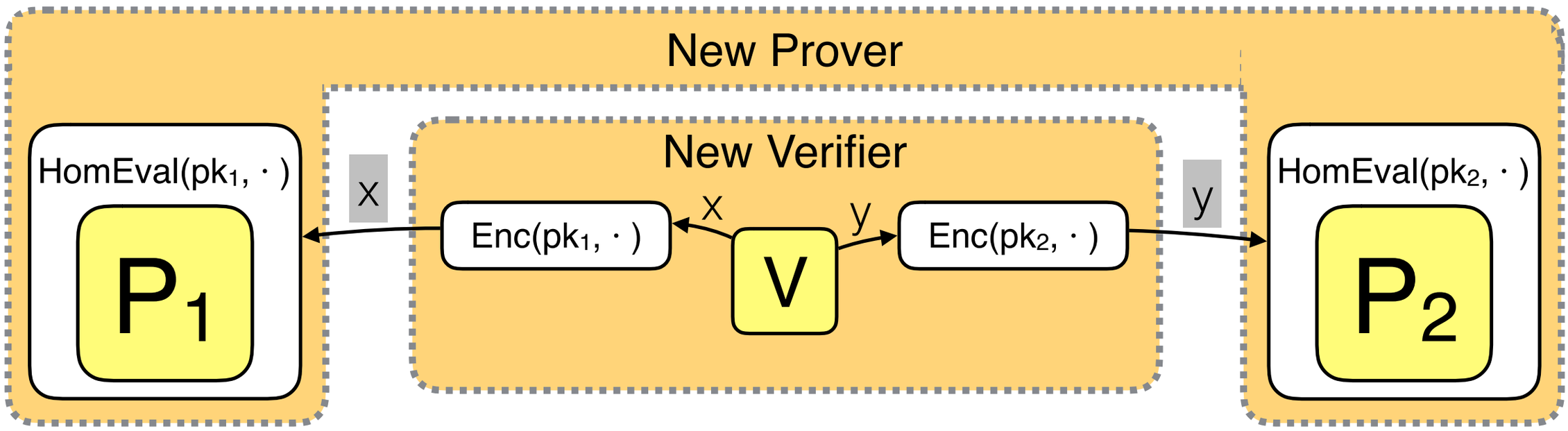
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P1
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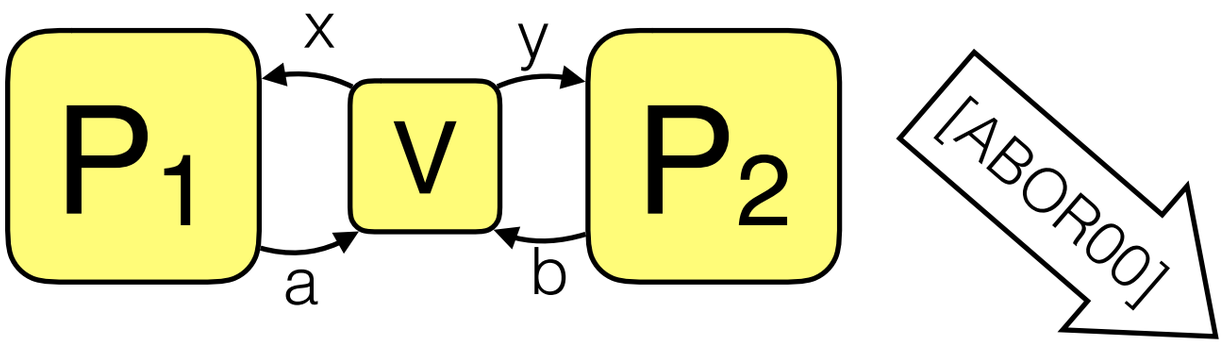
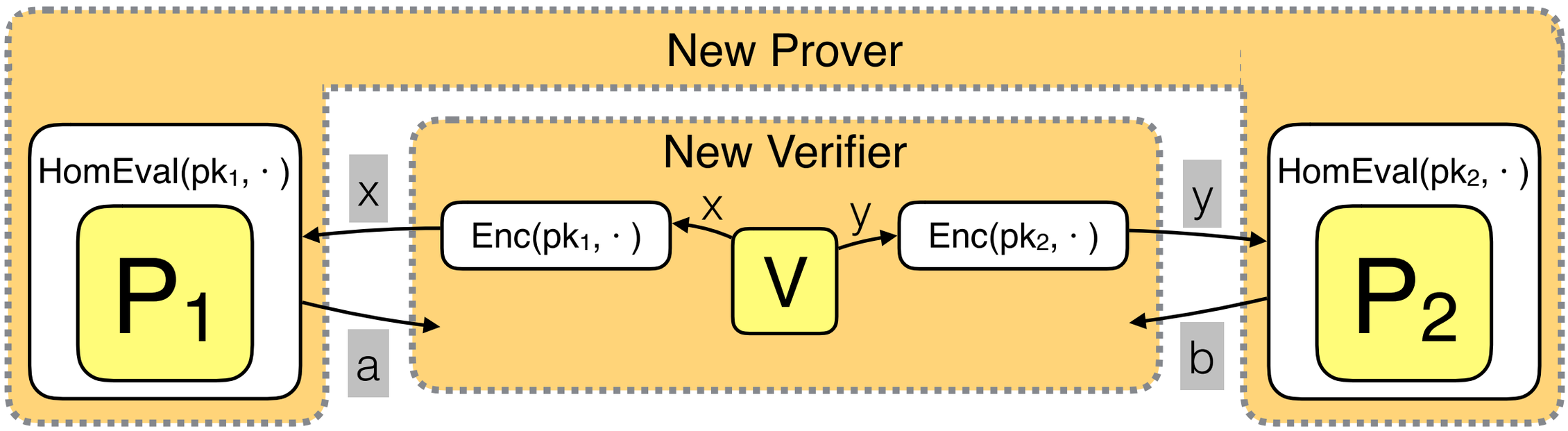
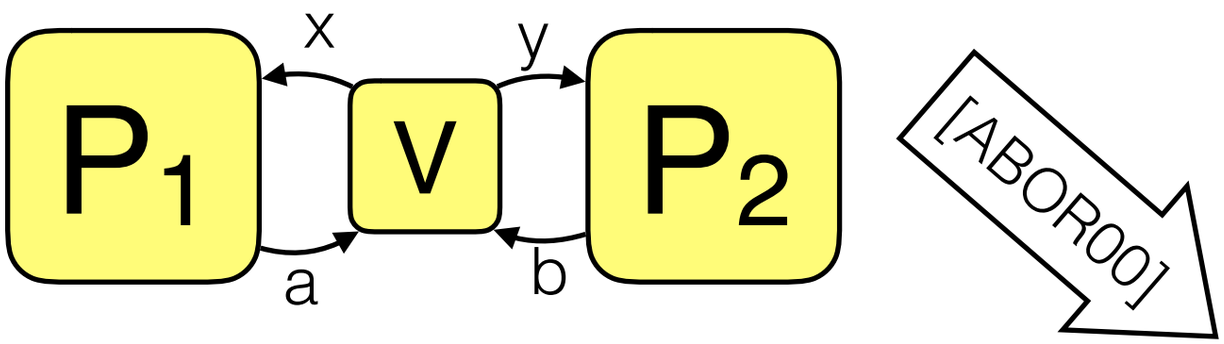
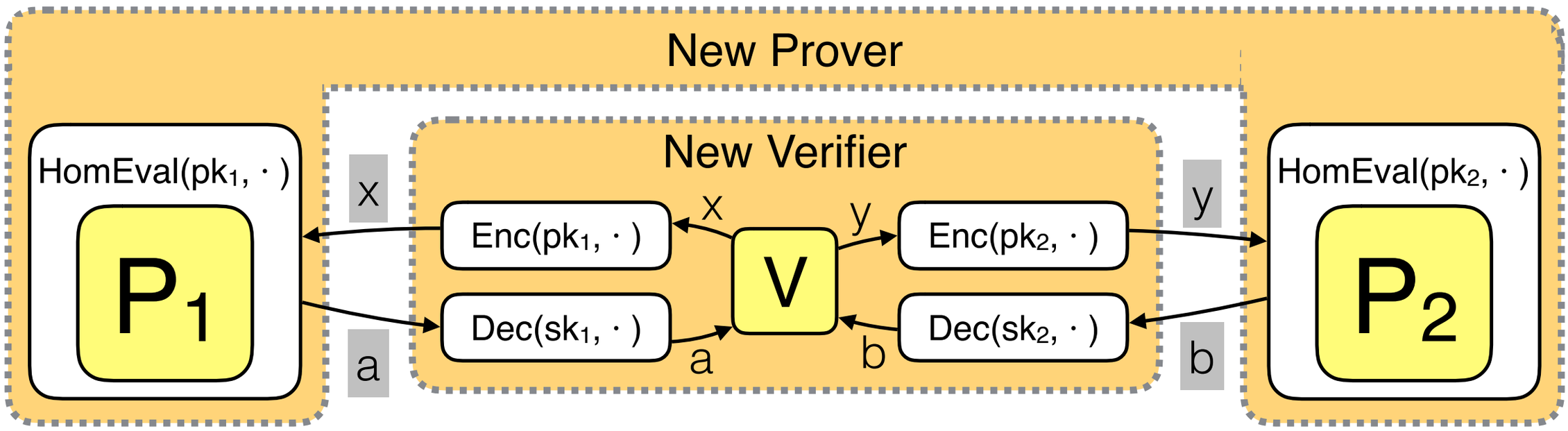

P1
V
P2
x
y
a
b

[ABOR00]

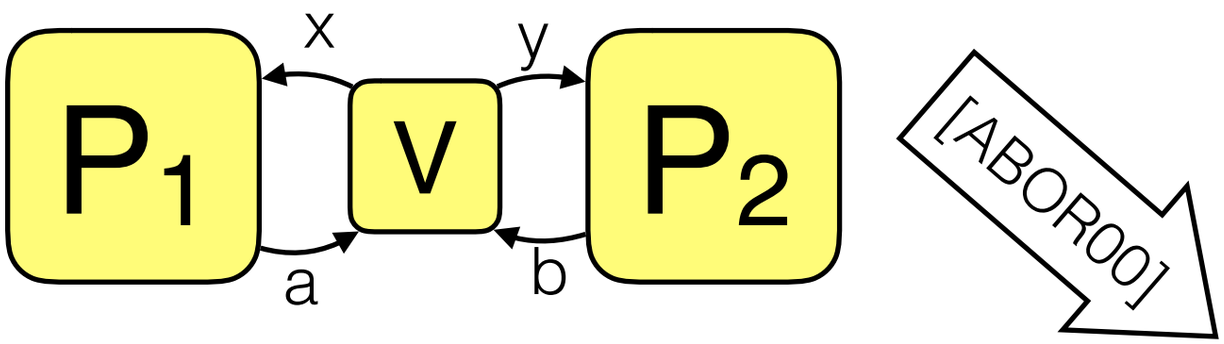

New Prover
New Verifier
HomEval(pk1, ·)
HomEval(pk2, ·)
Enc(pk1, ·)
Enc(pk2, ·)
x
y
V
P1
P2
a
b

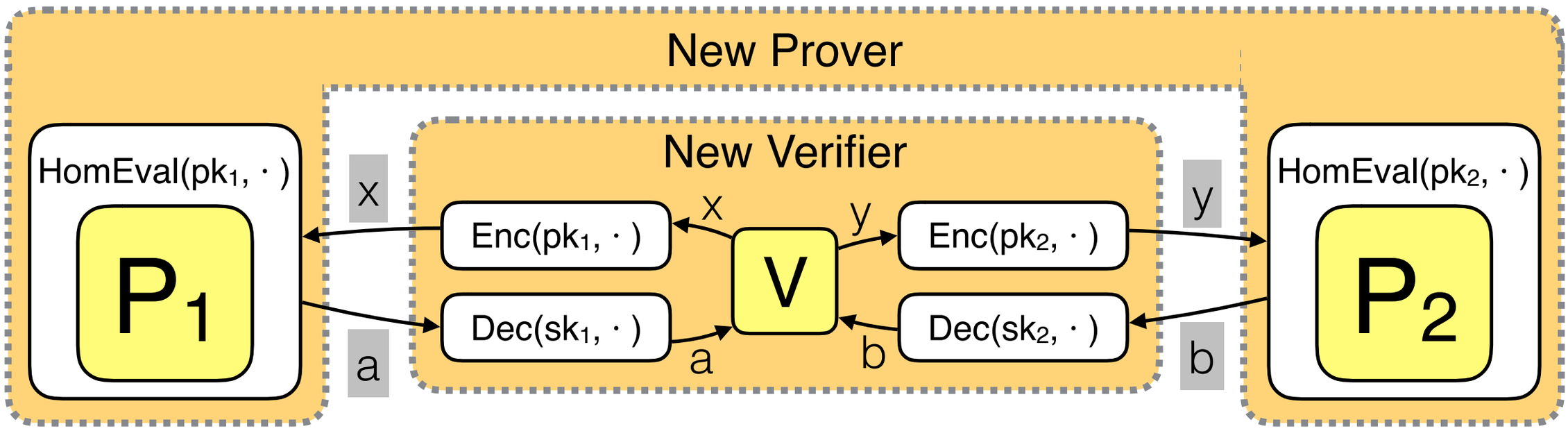
From **MIPs**


P1
V
P2
x
y
a
b
[ABOR00]


New Prover
New Verifier
HomEval(pk1, ·)
HomEval(pk2, ·)
P1
P2
V
Enc(pk1, ·)
Enc(pk2, ·)
Dec(sk1, ·)
Dec(sk2, ·)
x
y
a
b

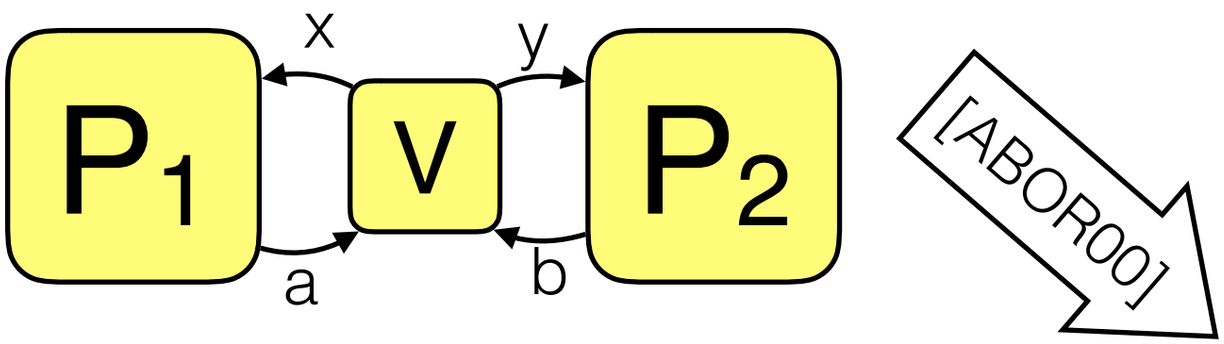
From **MIPs**

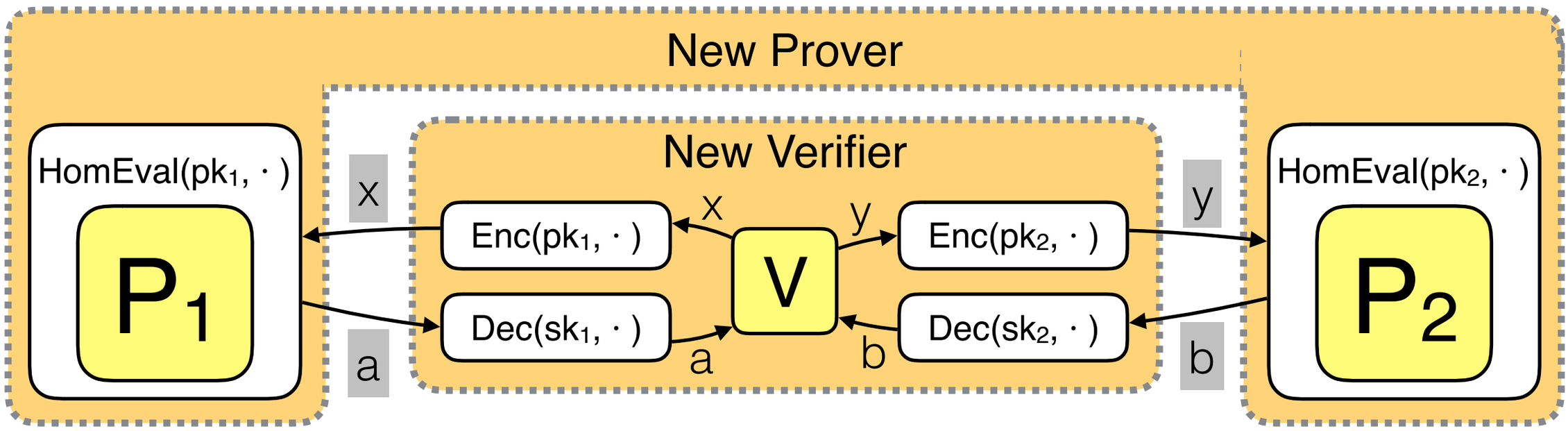




work	type of MIP	number of messages	cryptographic tool
[BC12]	standard (captures **NP**)	4	FHE
[BC12]		2	*extractable* FHE
[DHRW16]		2	*spooky-free* FHE
[KRR13] [KRR14]	no-signaling (captures **P**)	2	FHE

From **MIPs**





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From **Linear PCPs**

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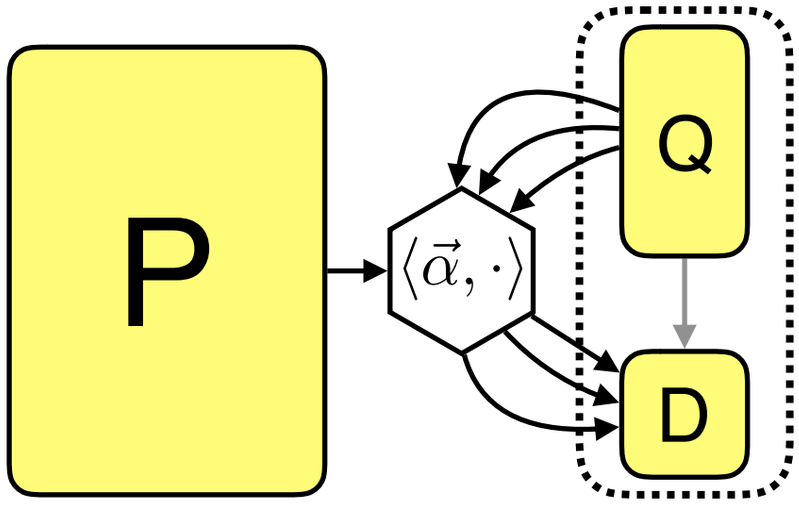
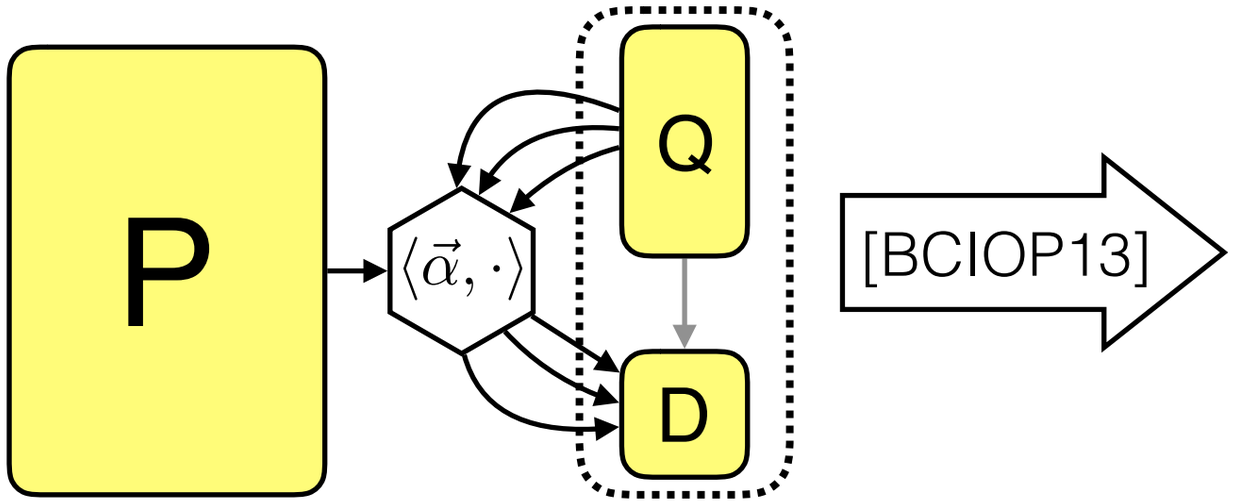
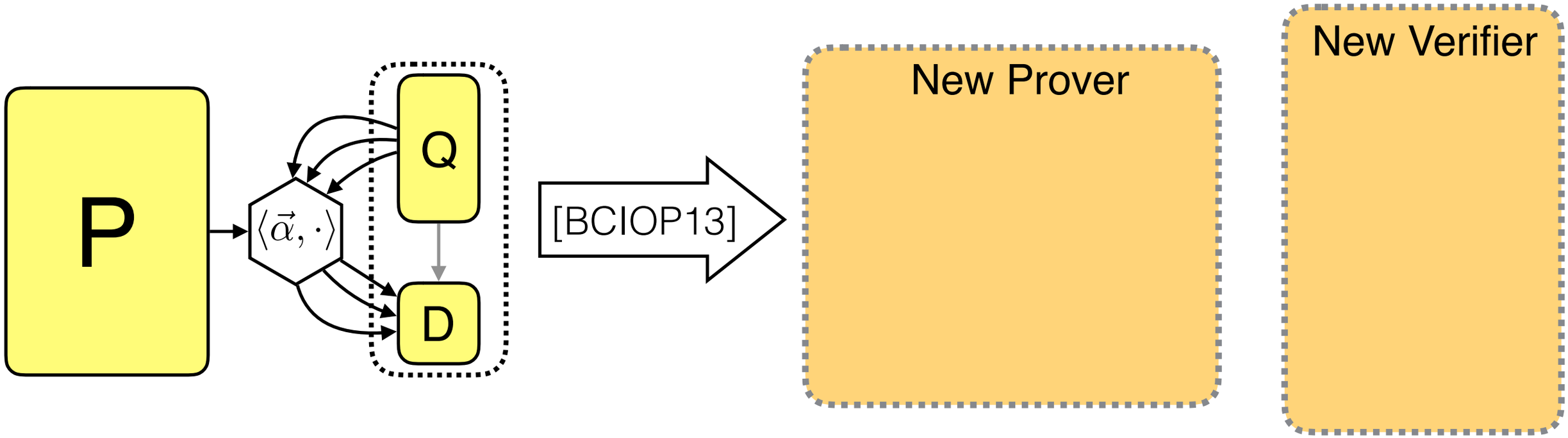


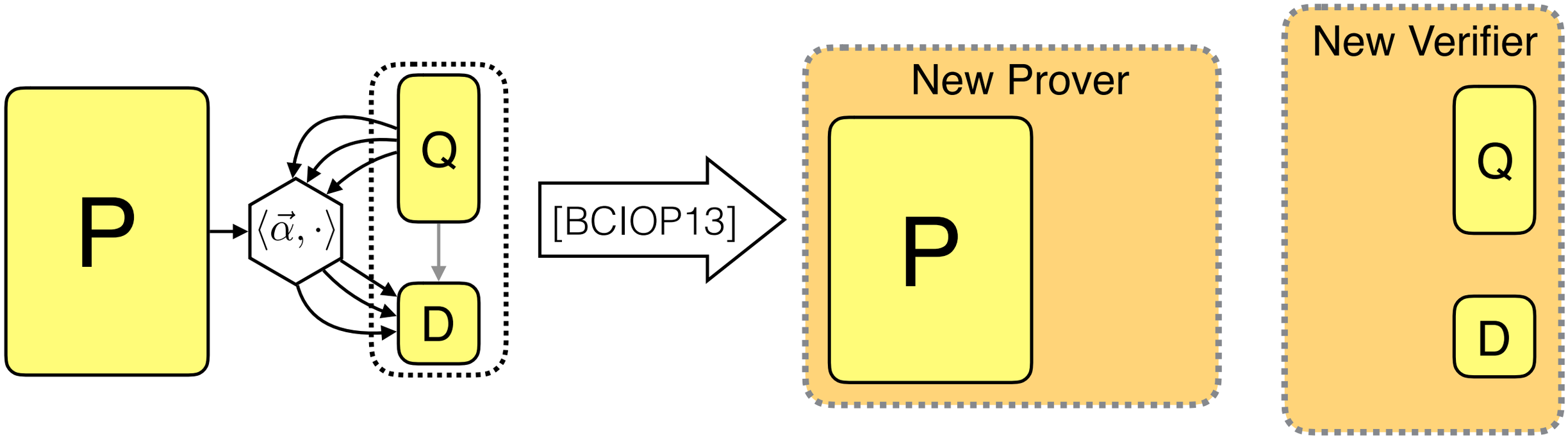
From **Linear PCPs**



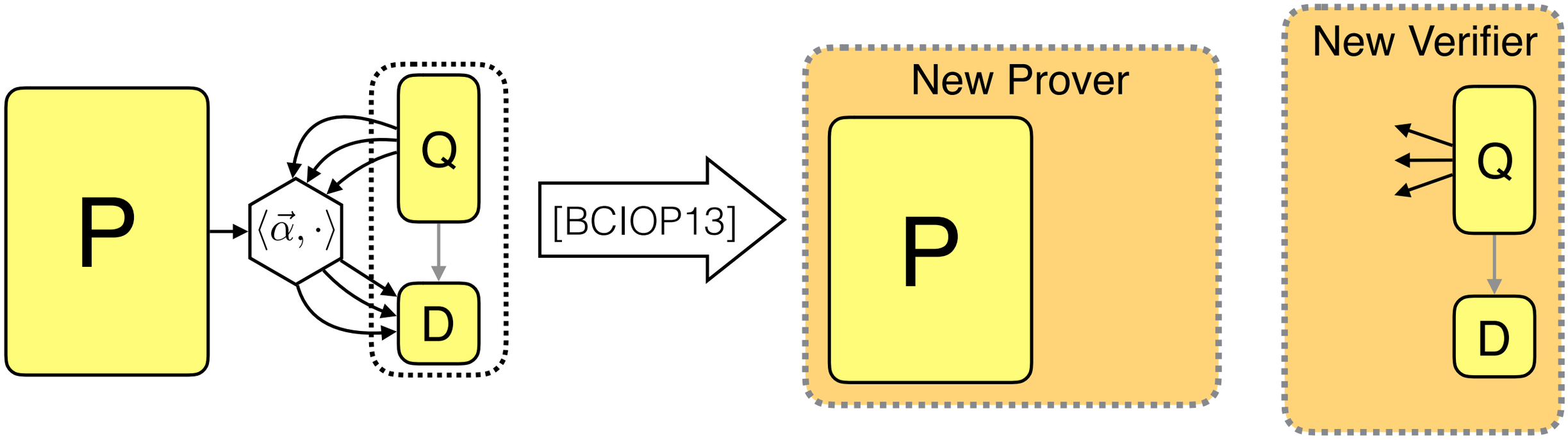
From **Linear PCPs**



From **Linear PCPs**



From **Linear PCPs**



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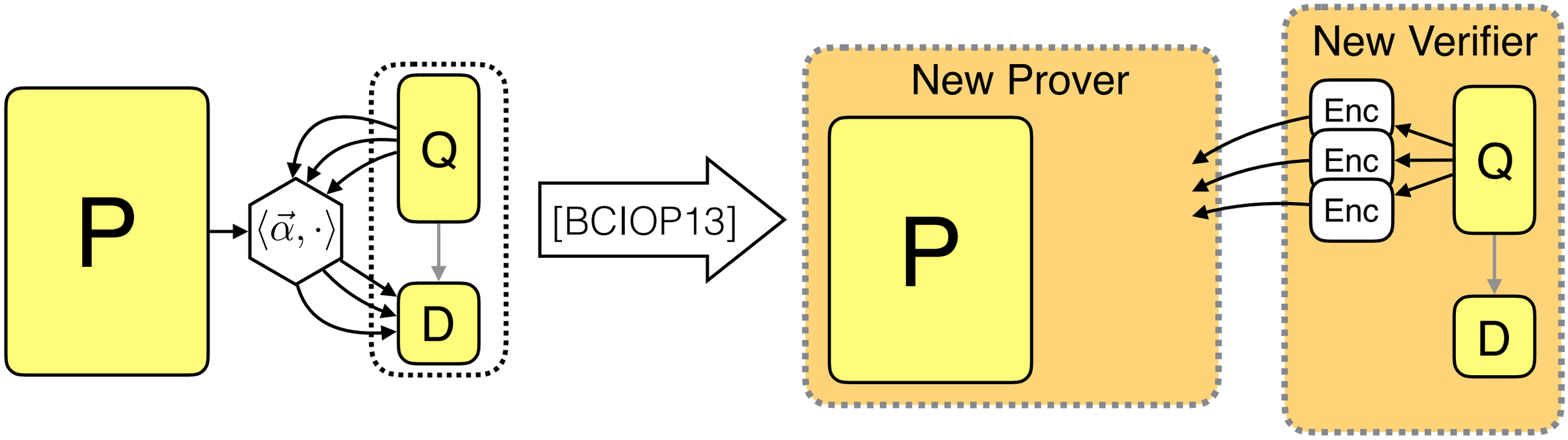

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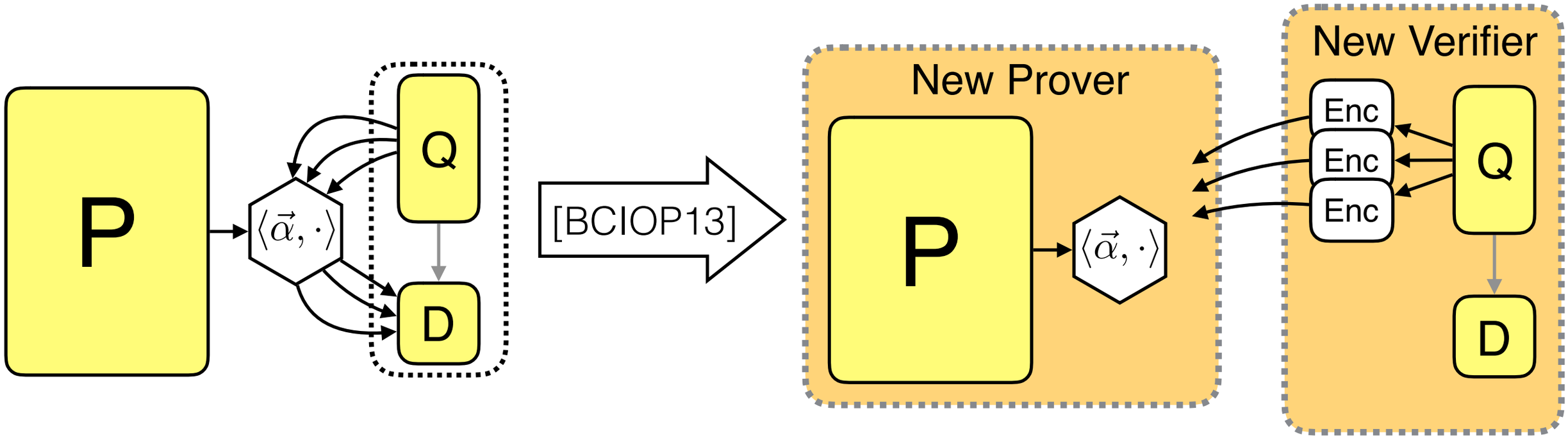

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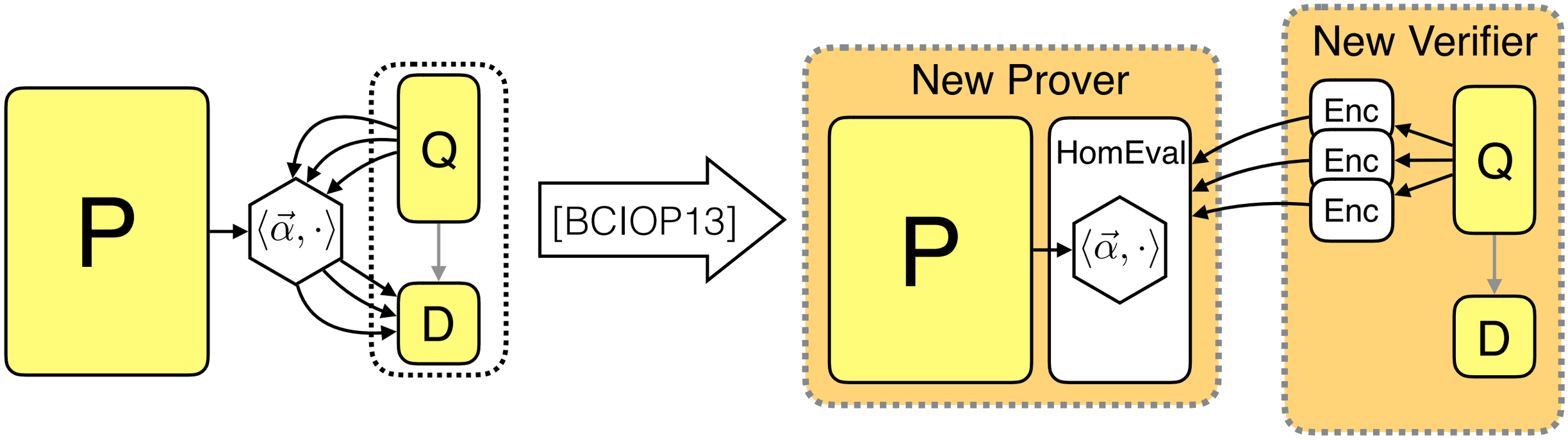

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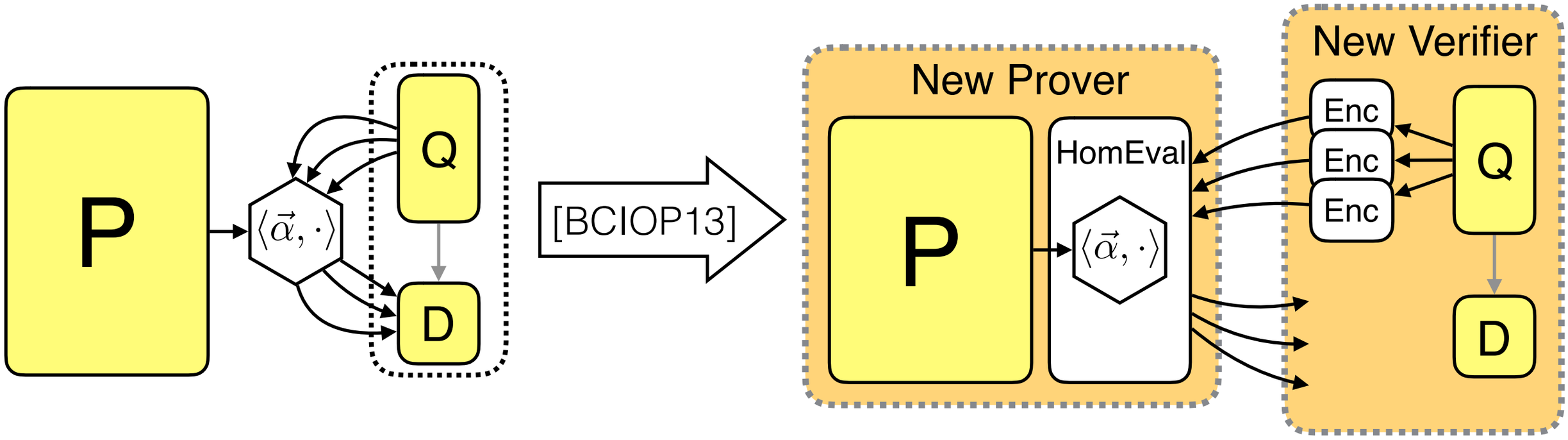

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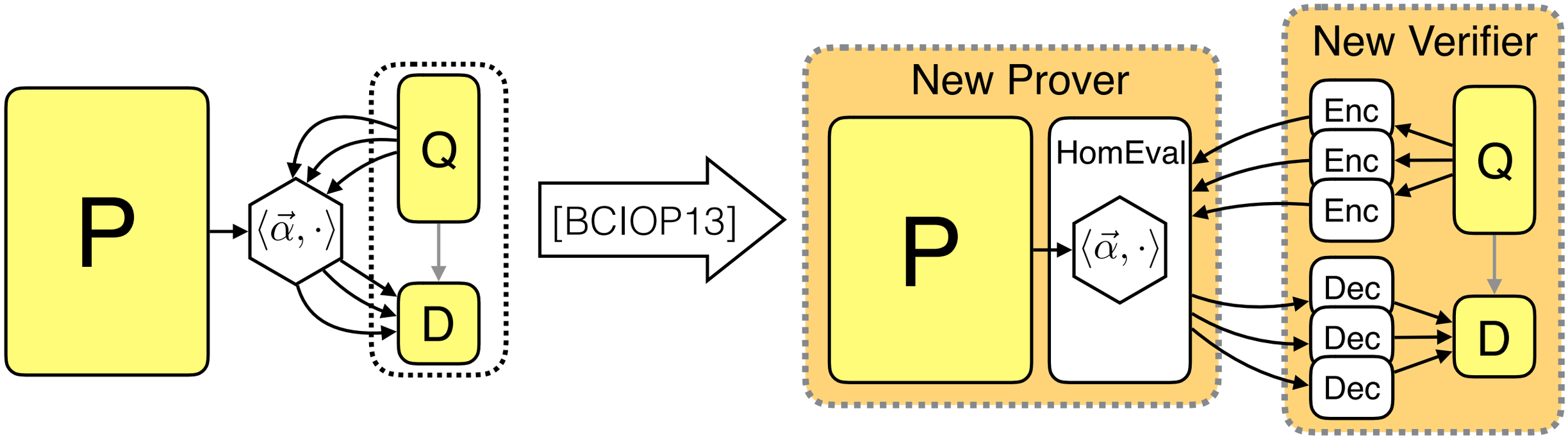

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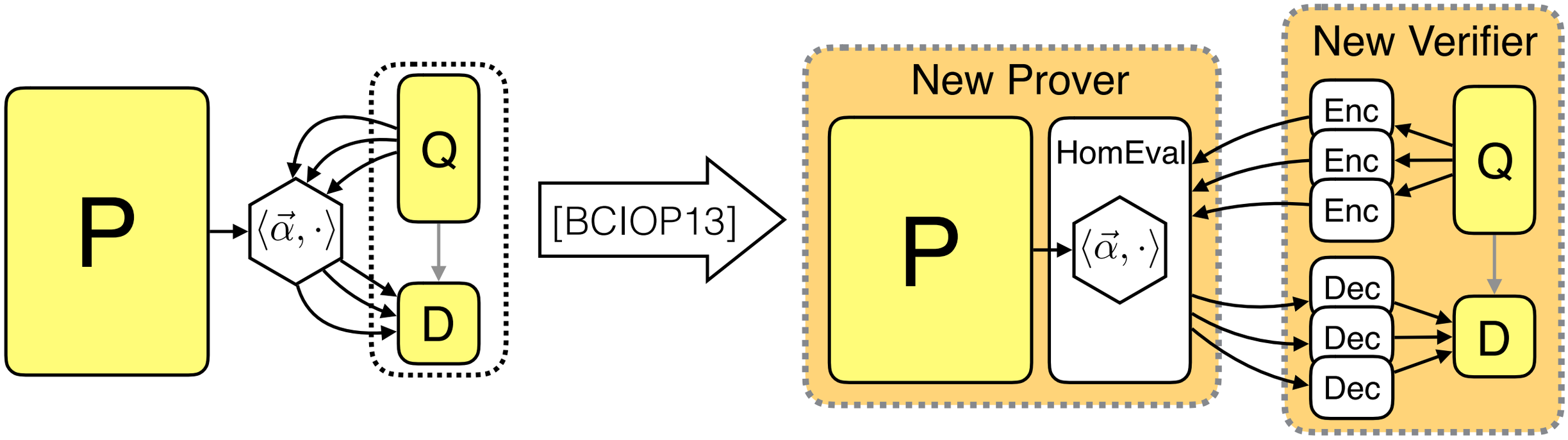

work	number of messages	cryptographic tool
[IKO07]	4	linearly-homomorphic encryption
[BCIOP13]	2	linear-only encryption
[BISW17]	2	linear-only vector encryption
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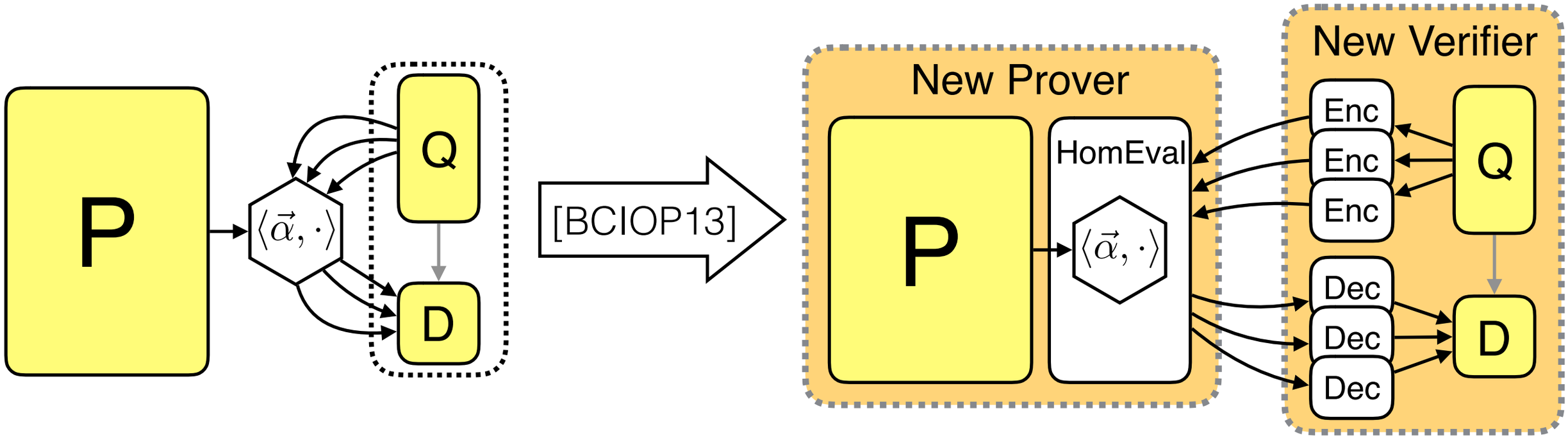

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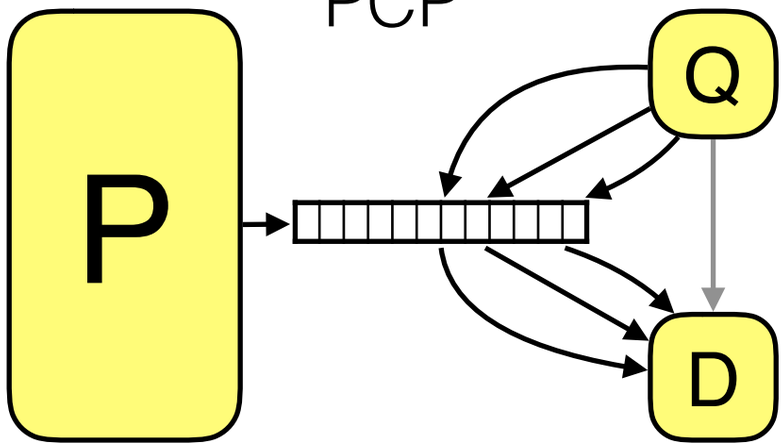
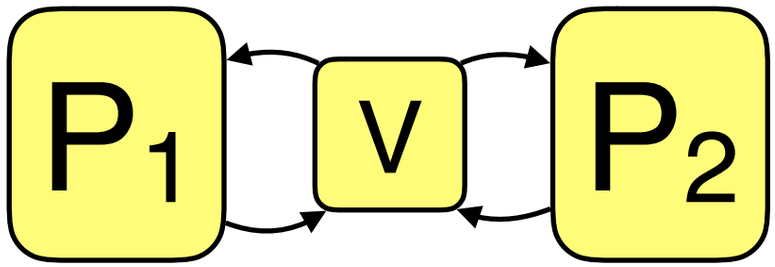
Tradeoffs

Tradeoffs

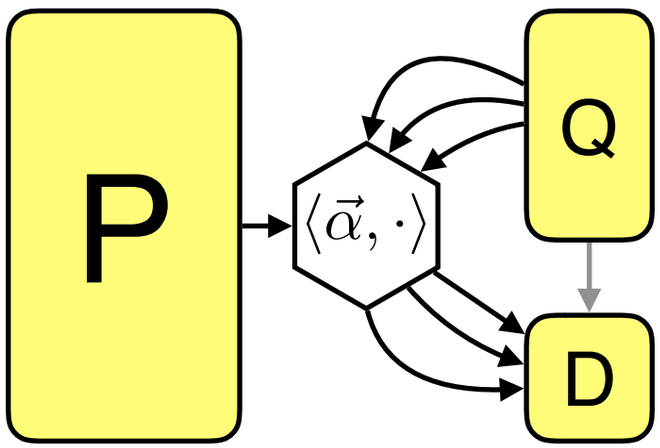
PCP



MIP



Linear PCP



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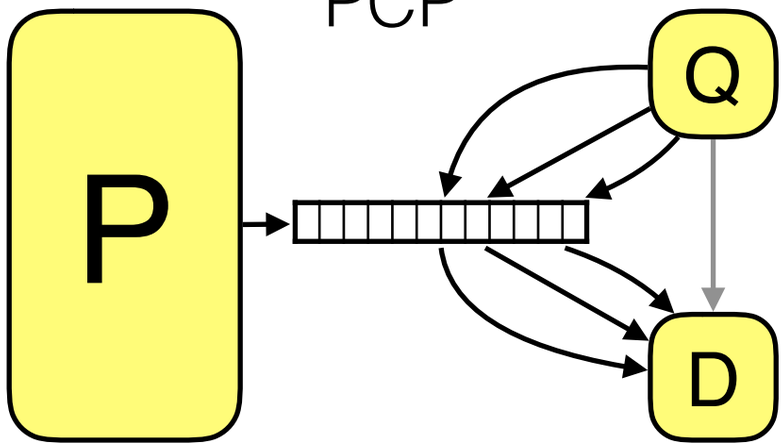
Tradeoffs

PCP



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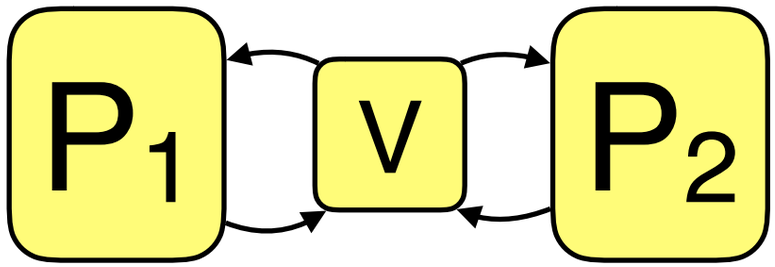

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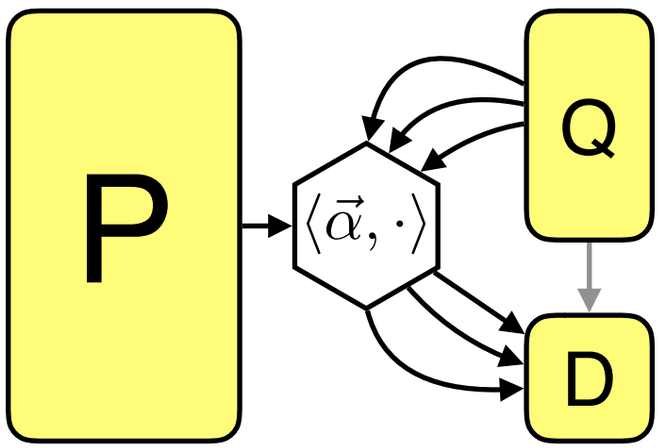
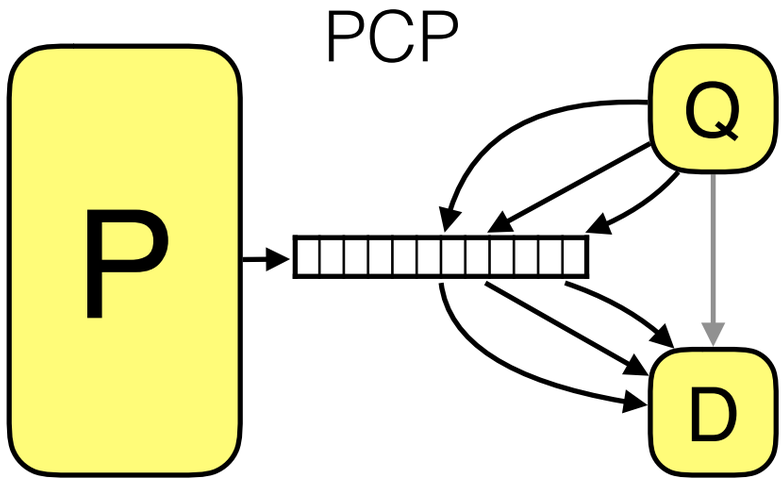
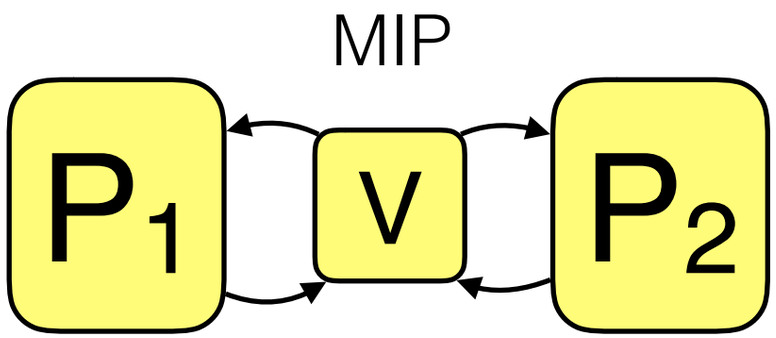


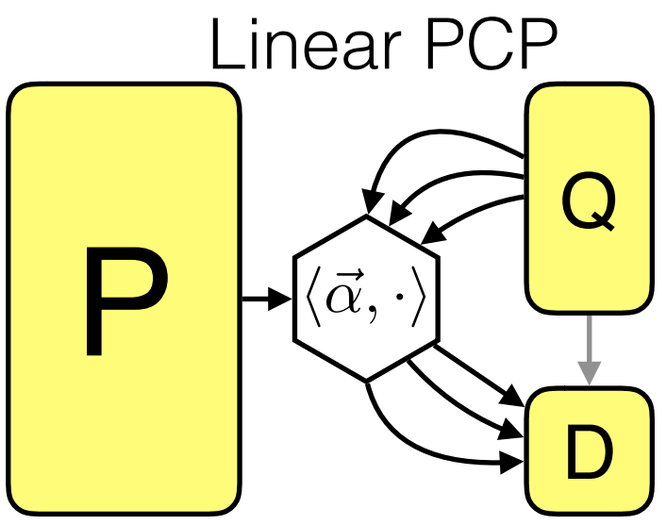
succinct arguments

...

Tradeoffs







...

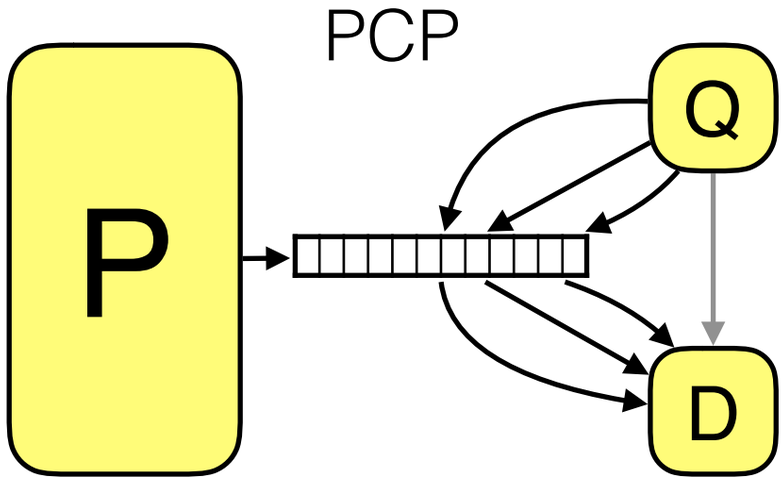
public verifiability

symmetric-key crypto

bottleneck is PCP

succinct arguments

Tradeoffs



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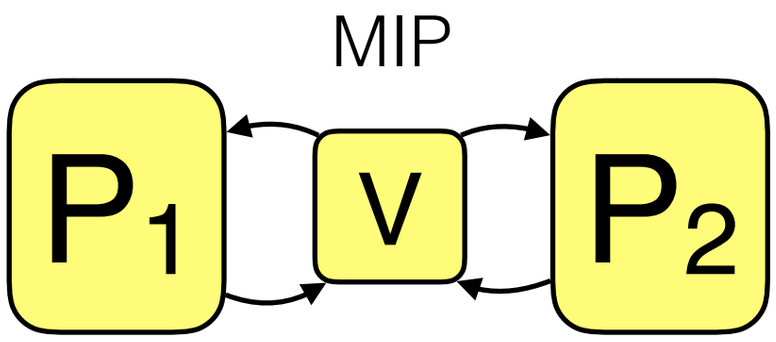

succinct arguments

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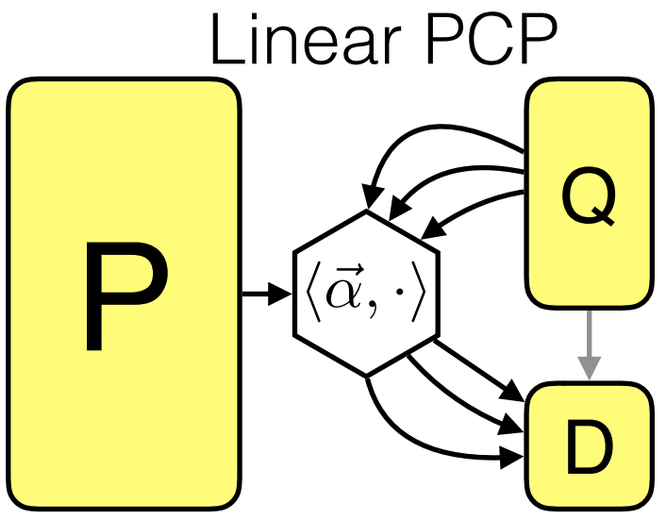
public-key crypto

bottleneck is MIP



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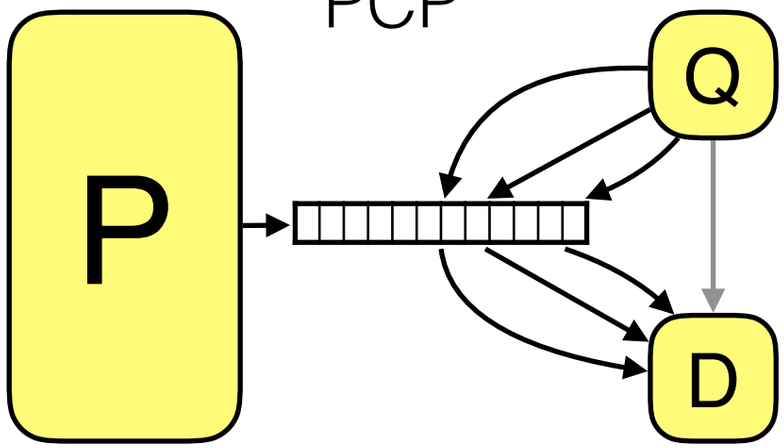
Tradeoffs

PCP



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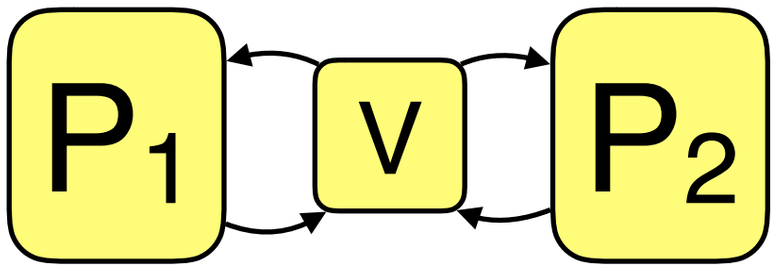
MIP



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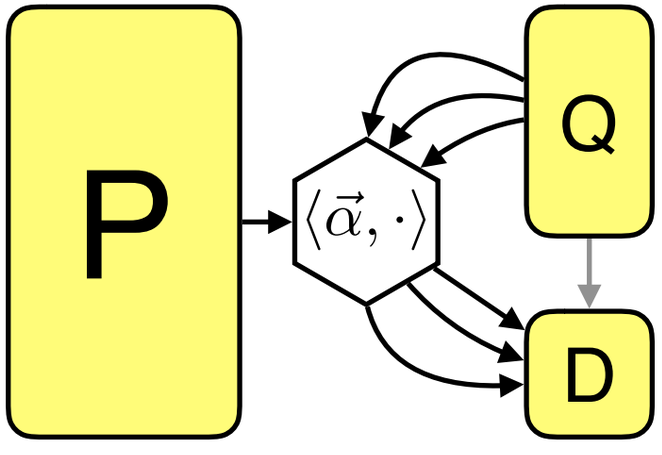


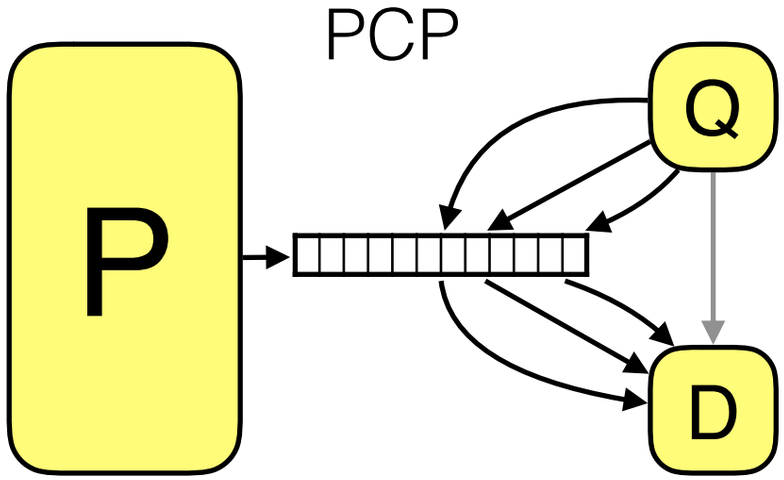
public/private verifiability

public-key crypto

quite efficient!!

...

Tradeoffs



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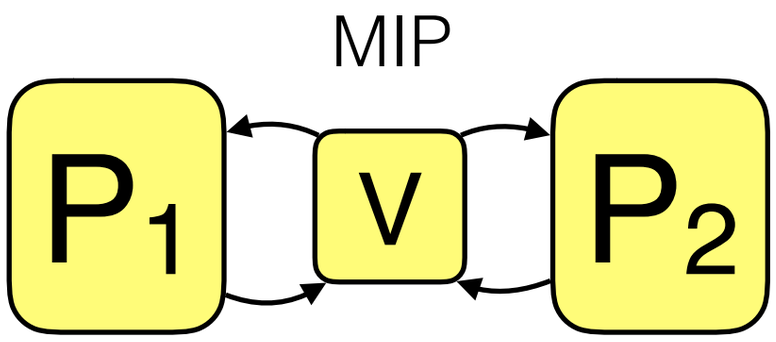

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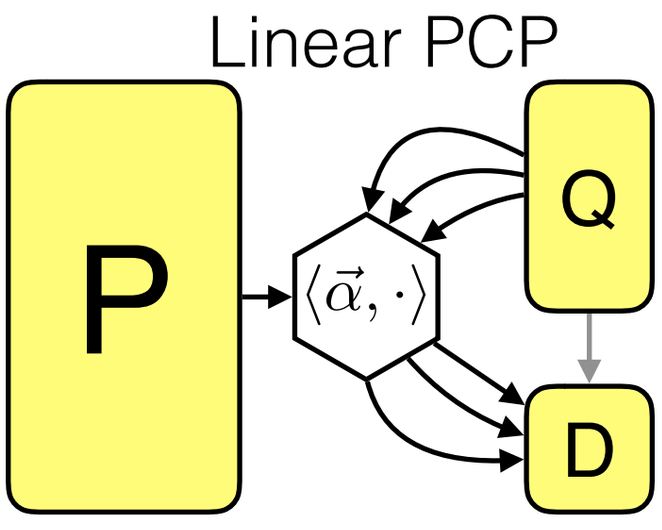

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|proof| ~ 128 bytes

|verifier| ~ 3ms

libsnark.org

...

Frontiers

Interactive Oracle Proofs

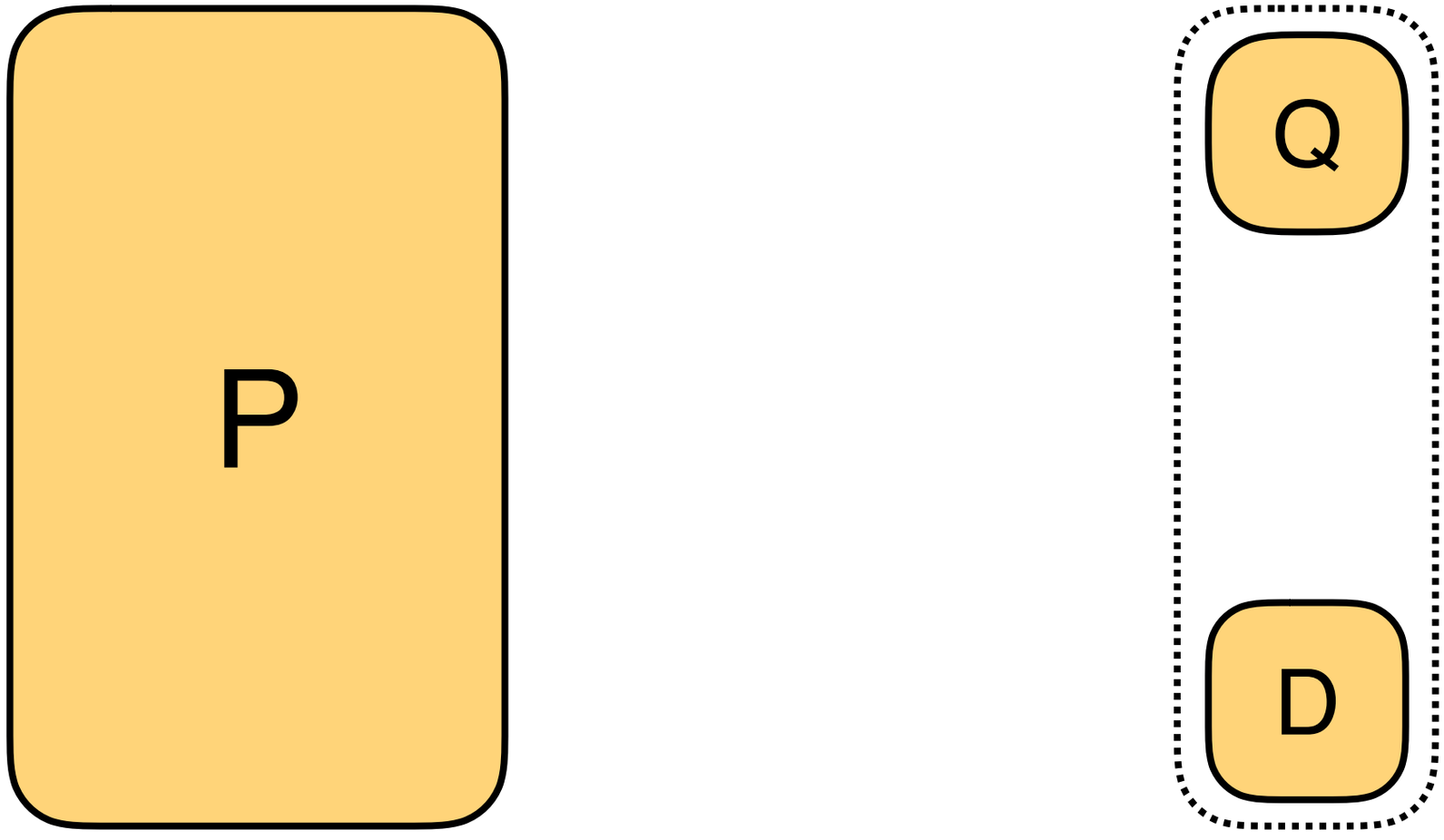
[BCS16][RRR16]

Probabilistically Checkable Interactive Proofs

Interactive Oracle Proofs

[BCS16][RRR16]

Probabilistically Checkable Interactive Proofs



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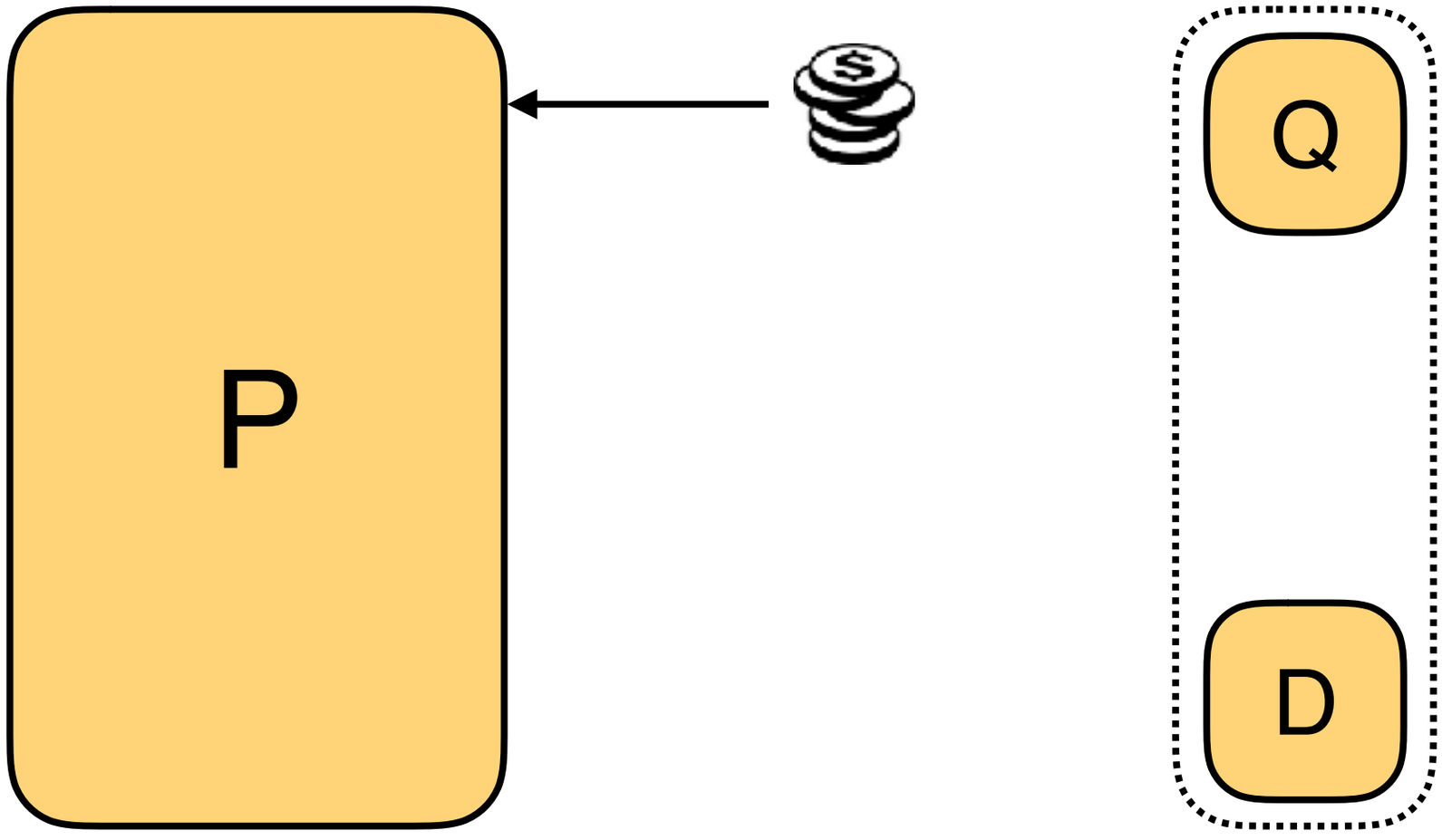

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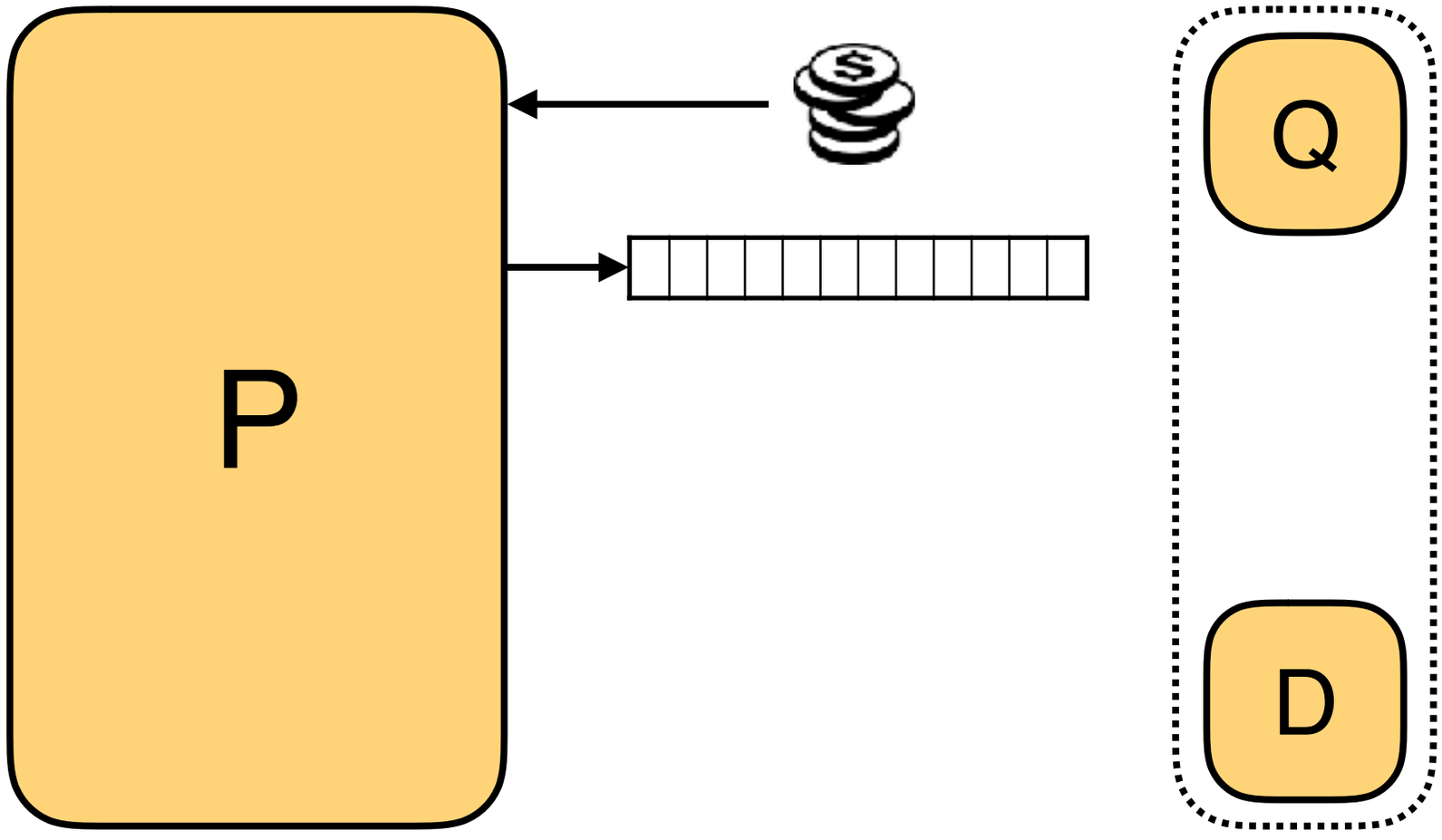

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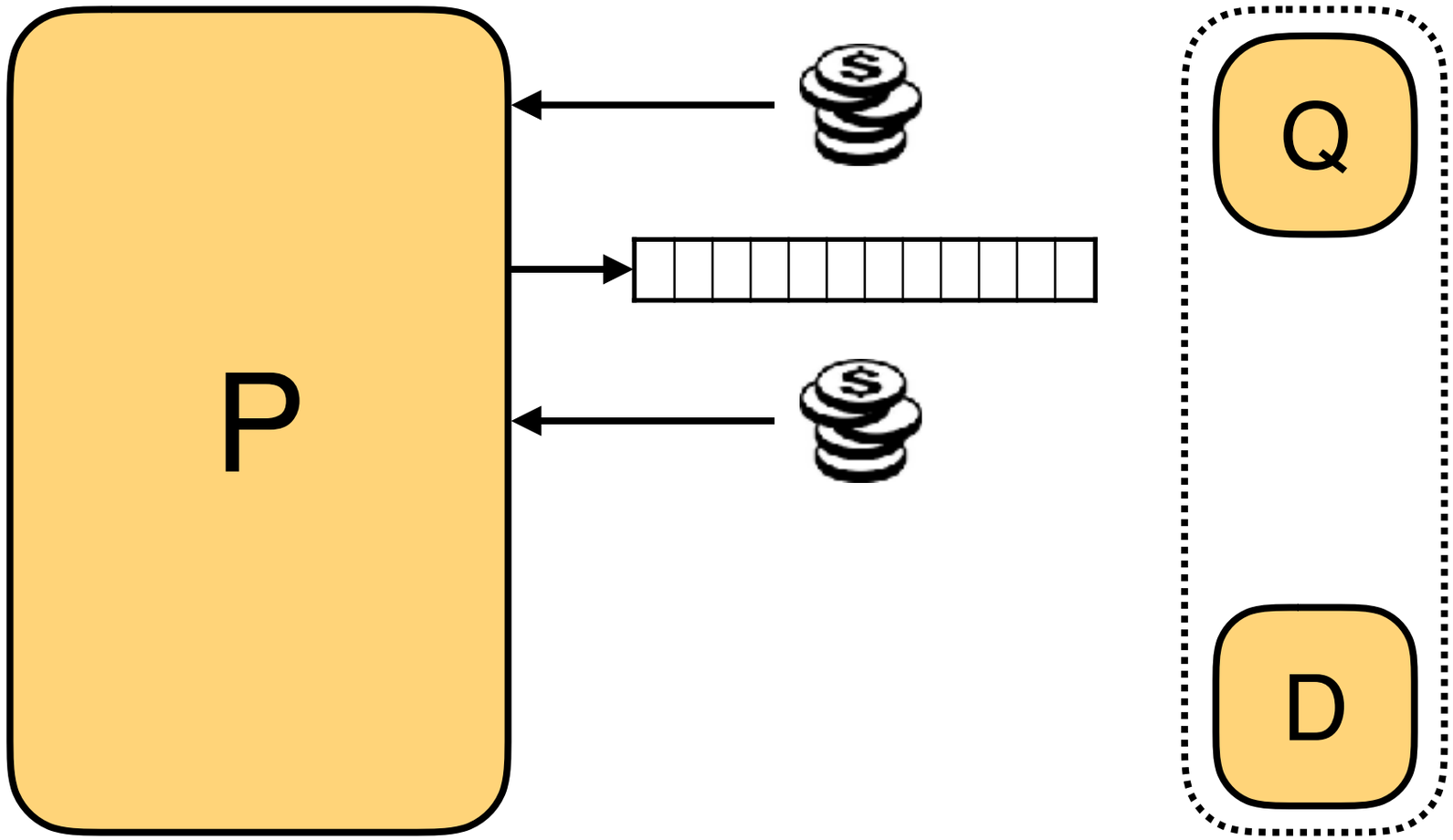

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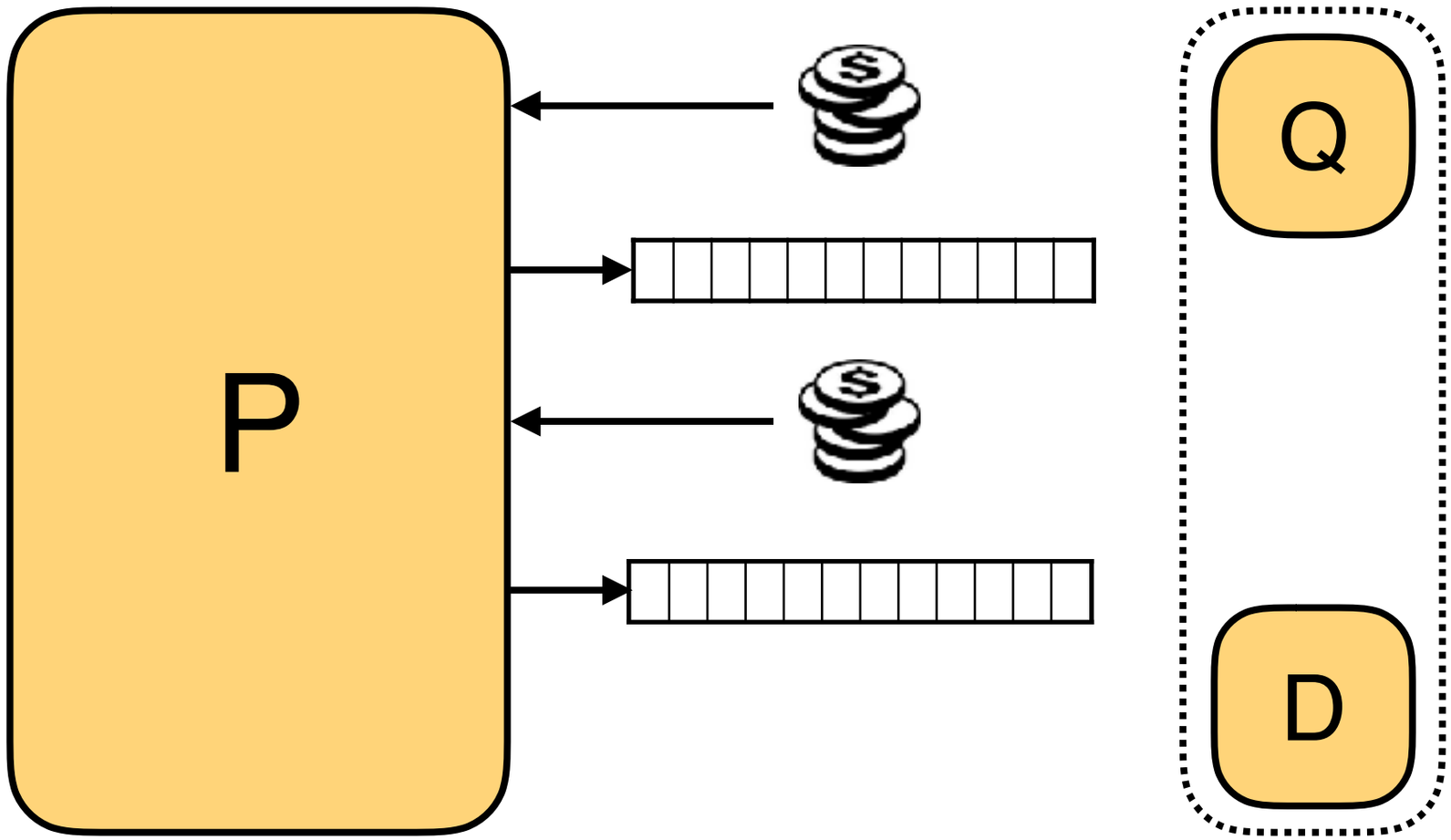

Interactive Oracle Proofs

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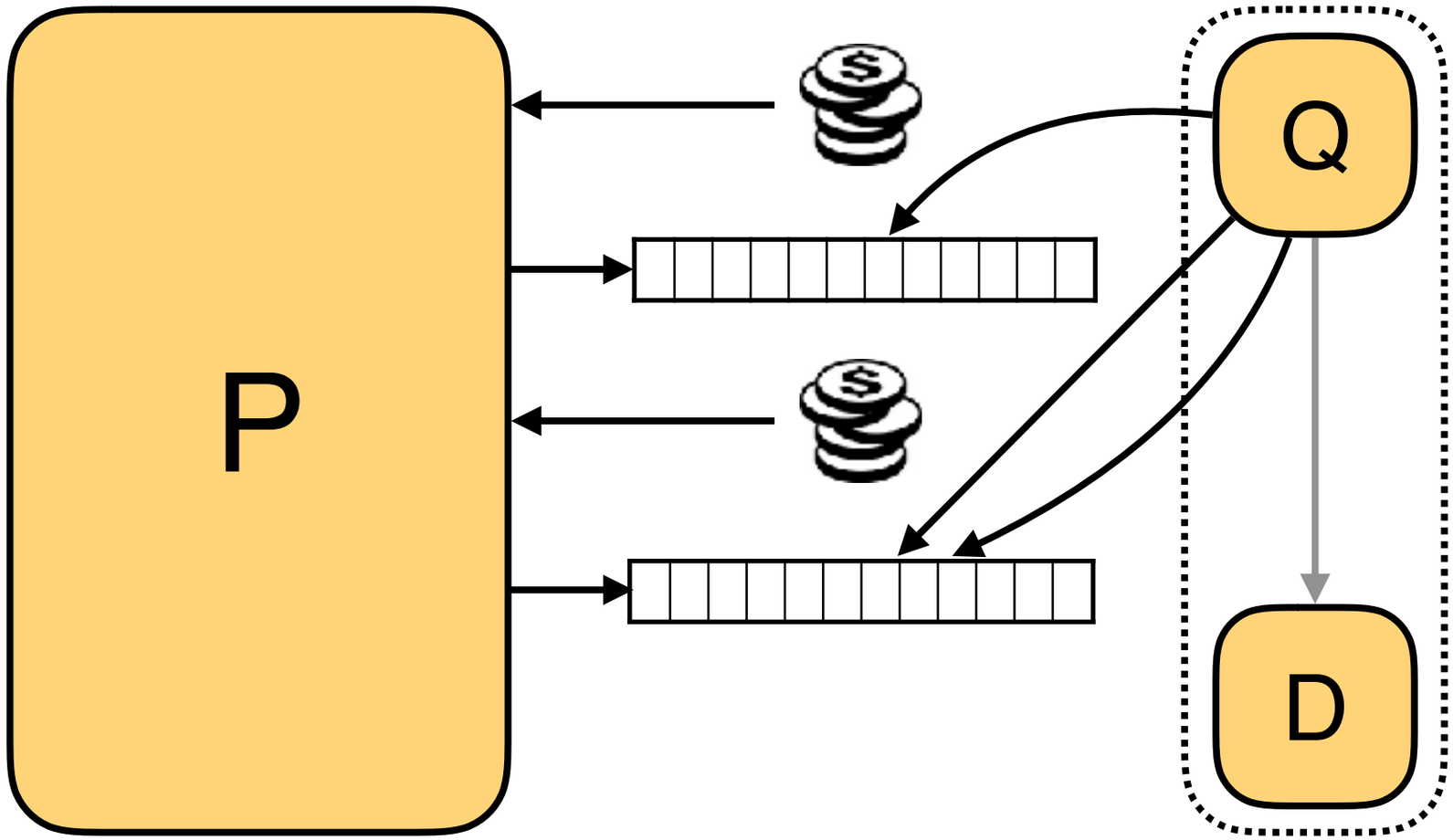

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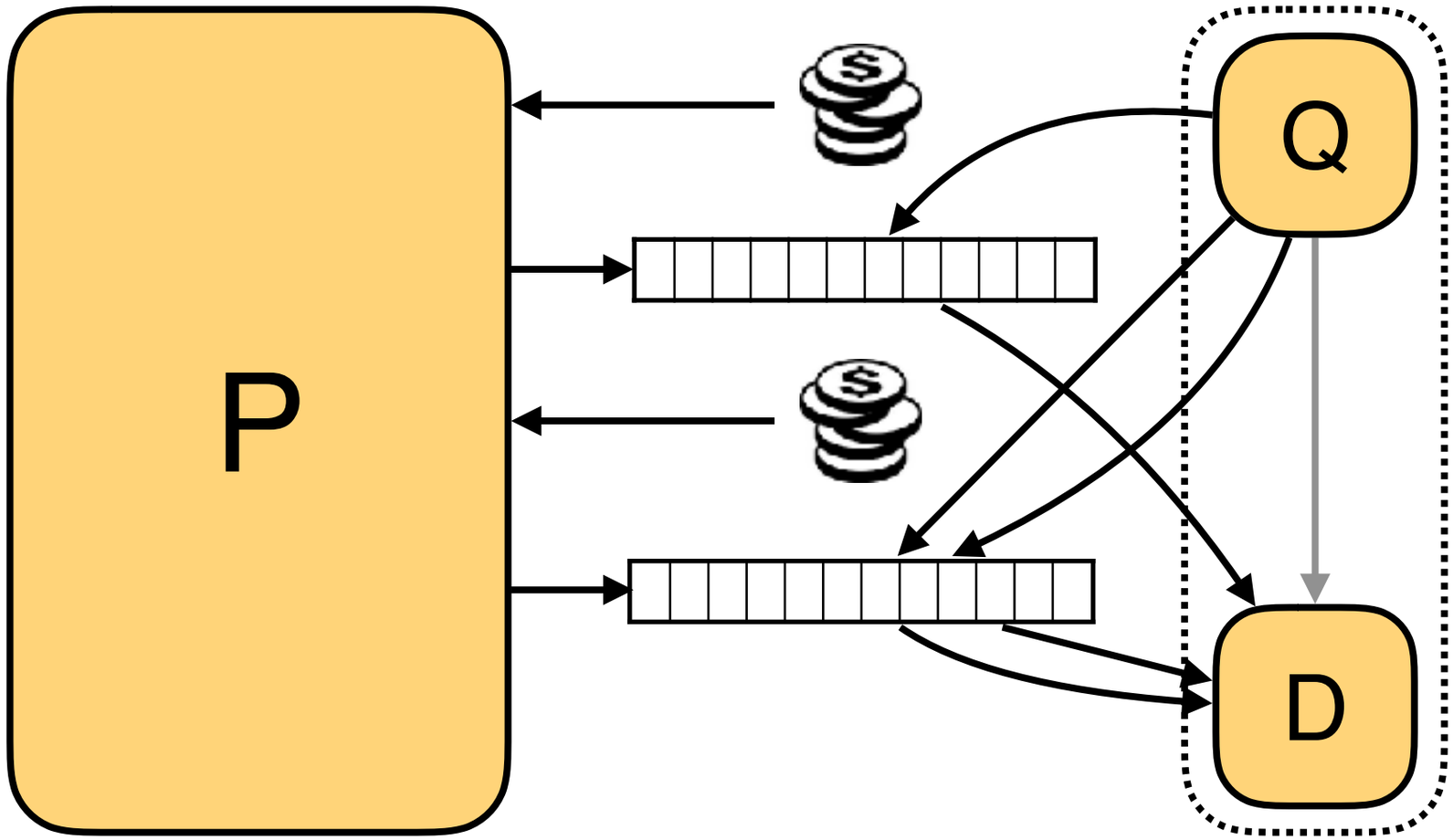
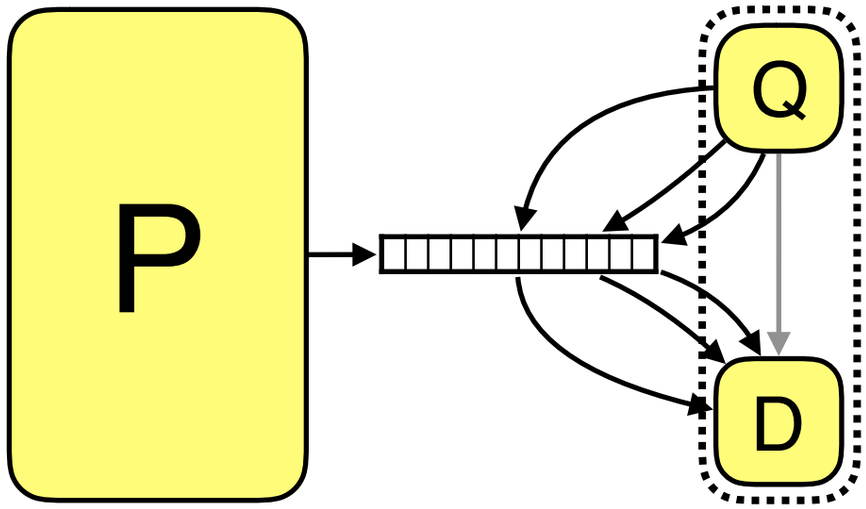


The verifier can simultaneously leverage randomness, interaction, and probabilistic checking.

Succinct Arguments From IOPs

Probabilistically Checkable Proof



Succinct Arguments From IOPs

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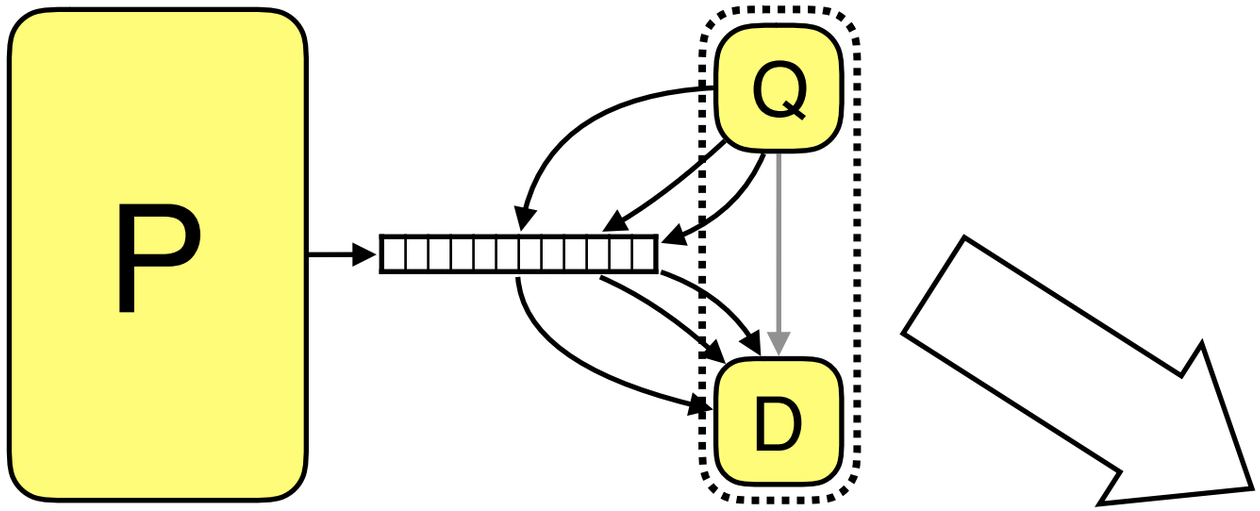

Succinct Argument

Succinct Arguments From IOPs

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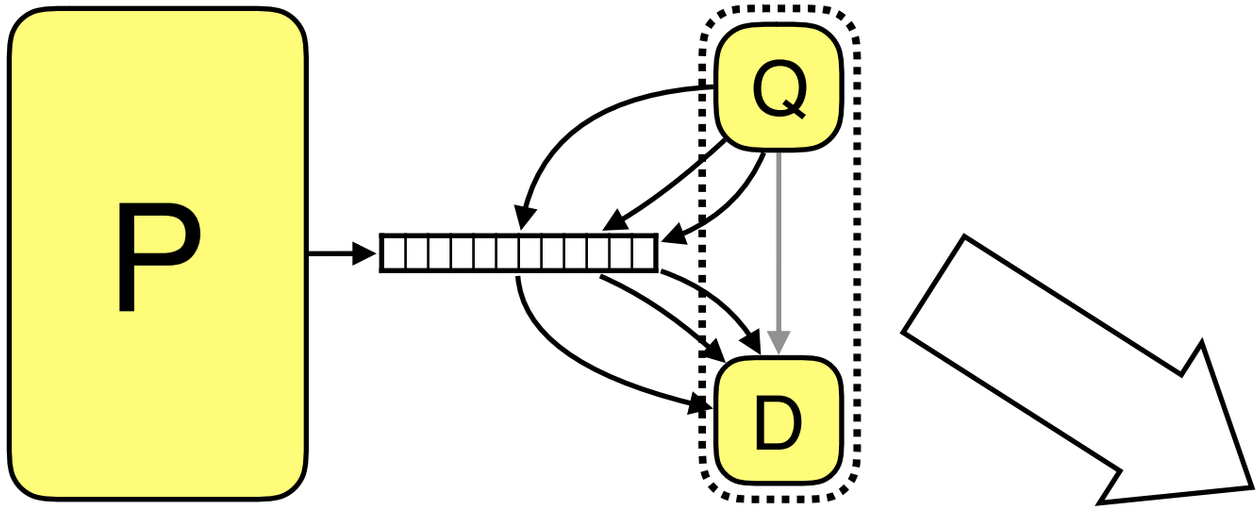
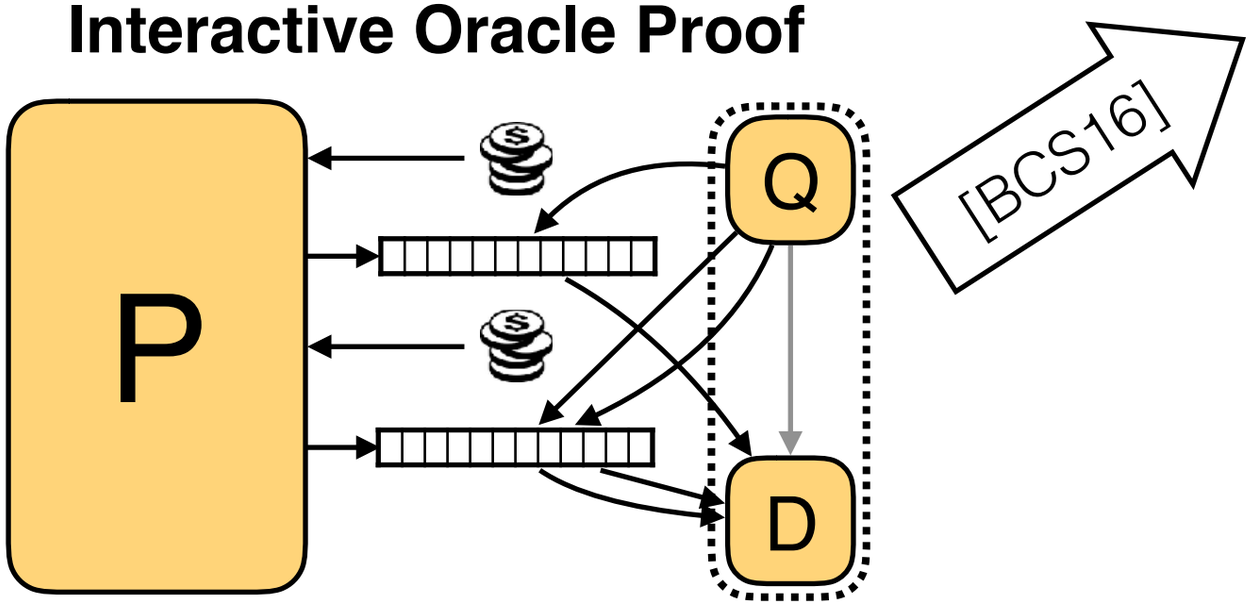


Succinct Argument

Interactive Oracle Proof



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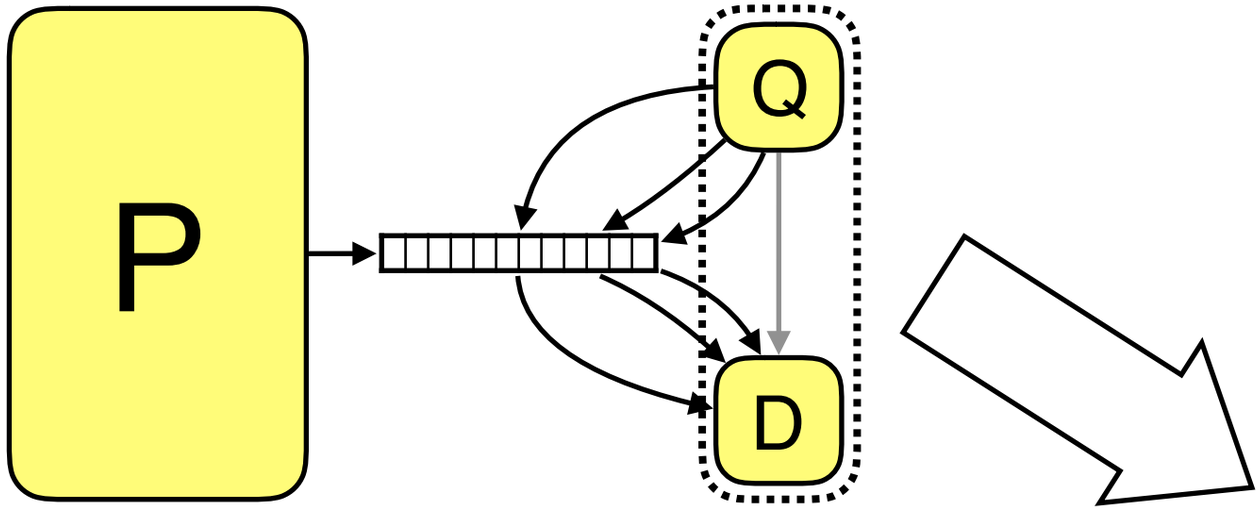

Succinct Argument

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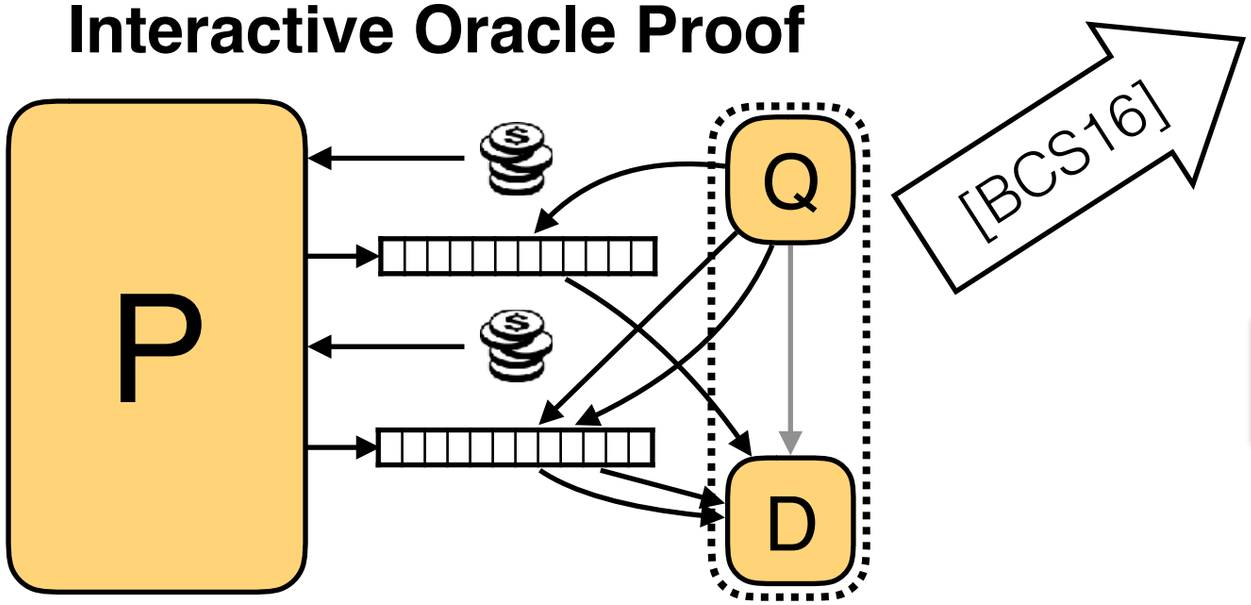
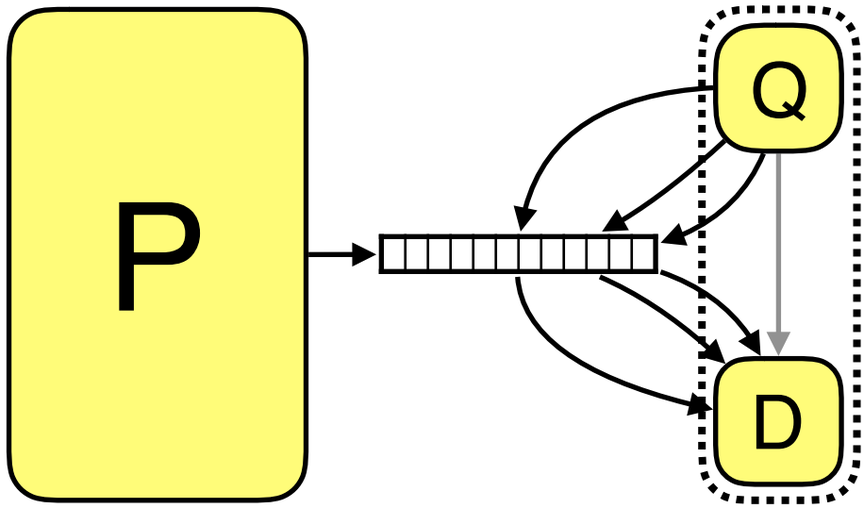
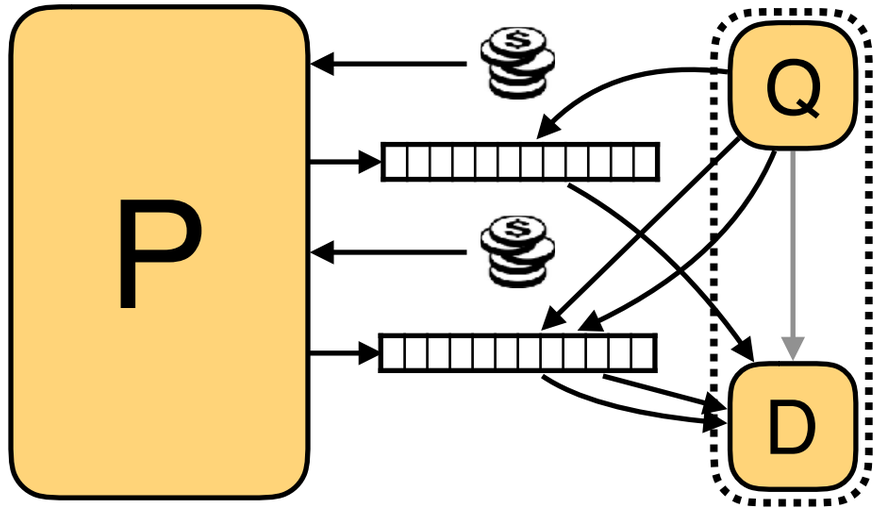


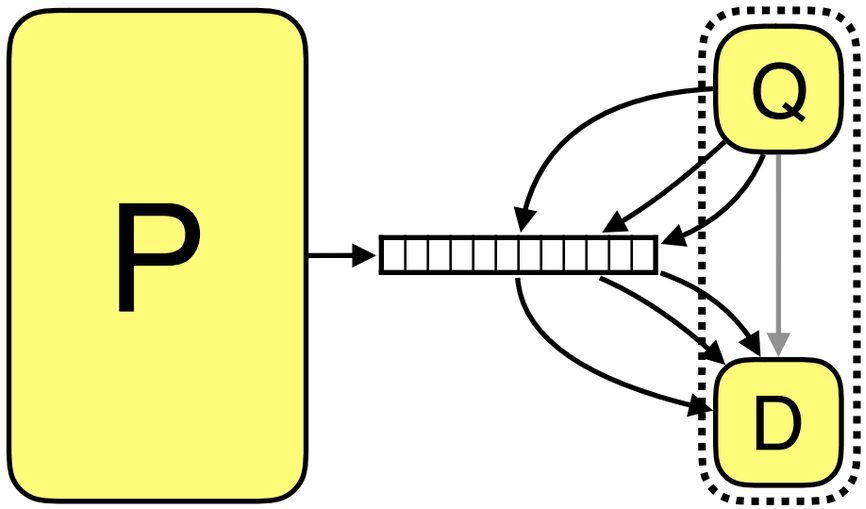
Q: any efficiency gains?

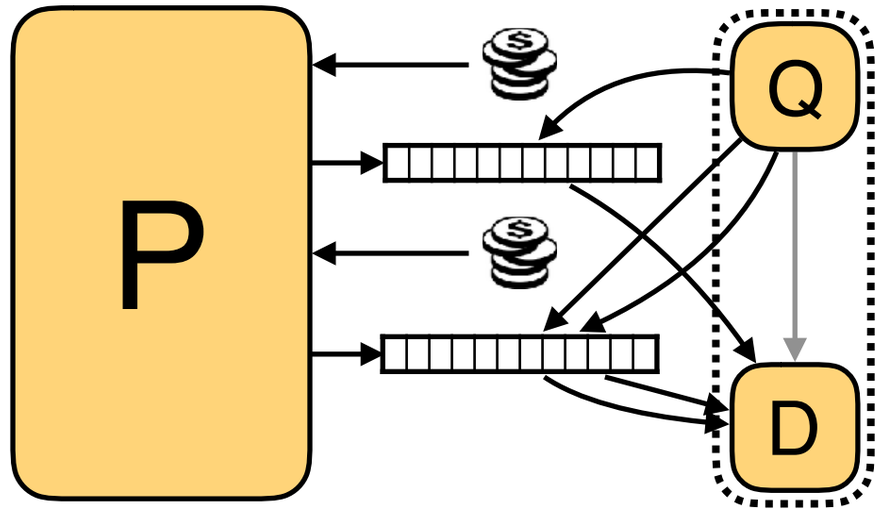
PCPs vs IOPs





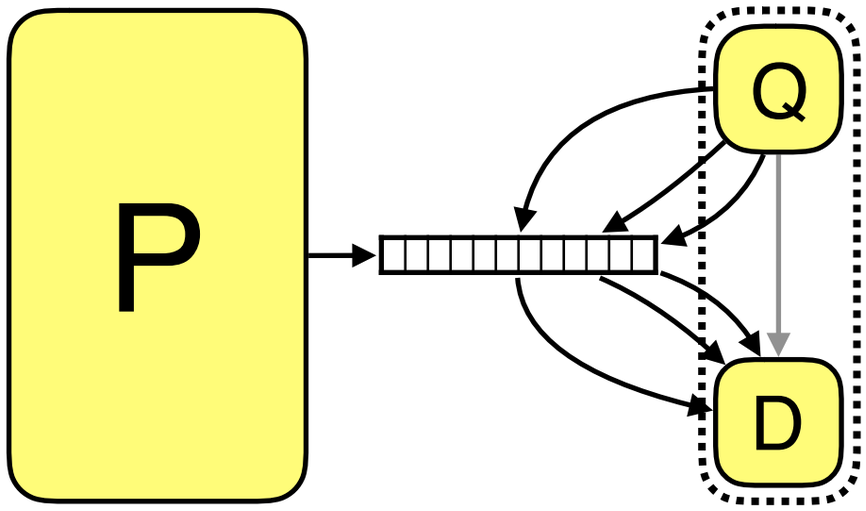
PCPs vs IOPs

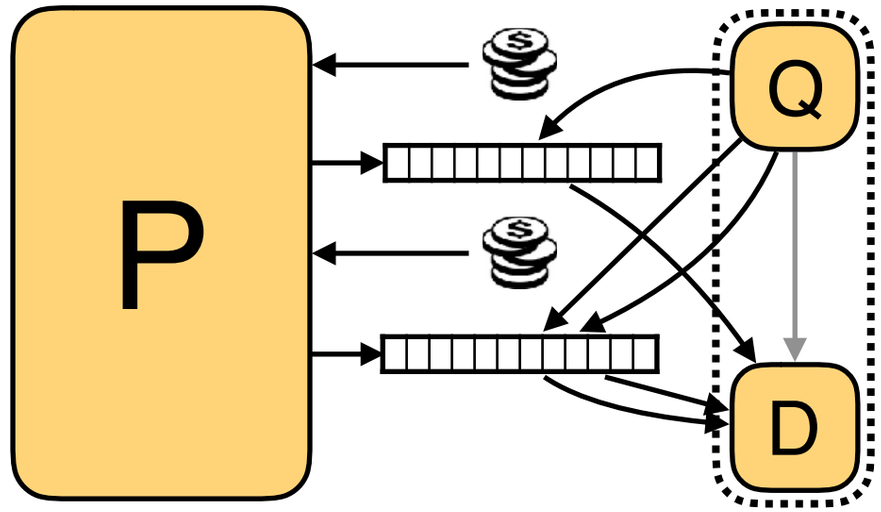




Best proof length vs query complexity tradeoff?

PCPs vs IOPs





Best proof length vs query complexity tradeoff?

Thm [BS08][Din07][Mie09]

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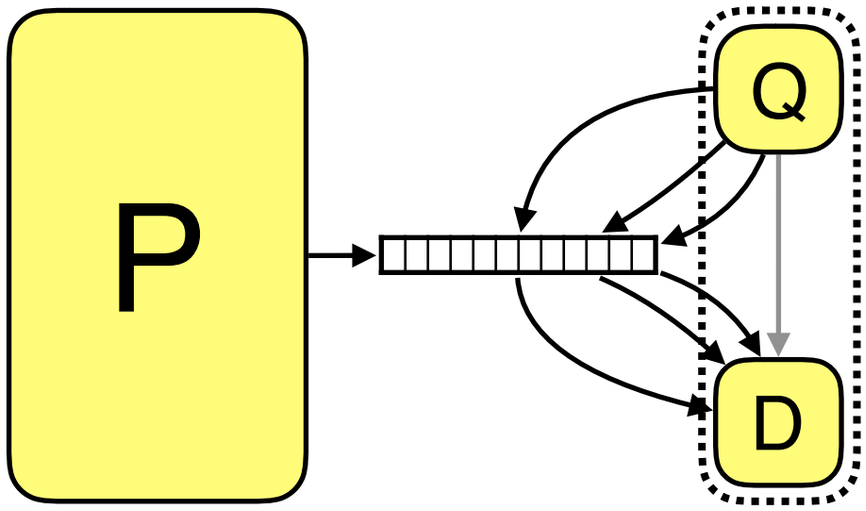
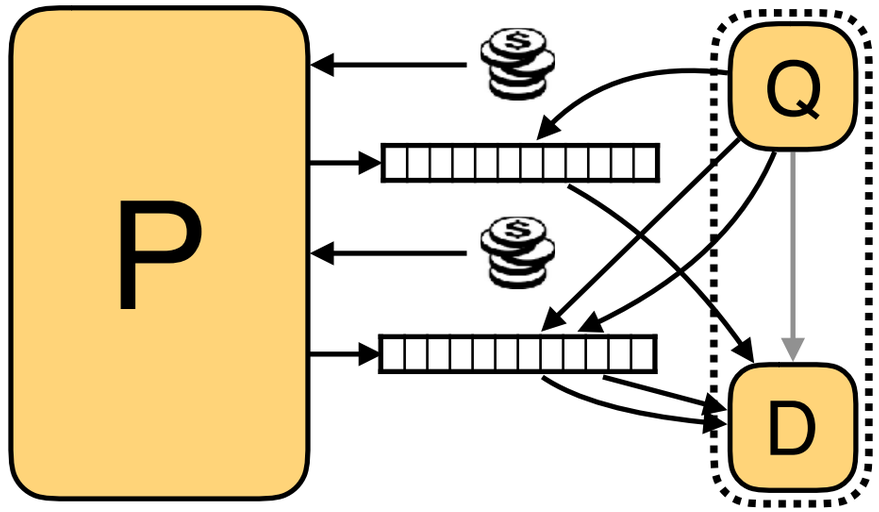
PCPs vs IOPs





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$\forall\ \varepsilon>0$ circuit SAT has (non-uniform) PCPs
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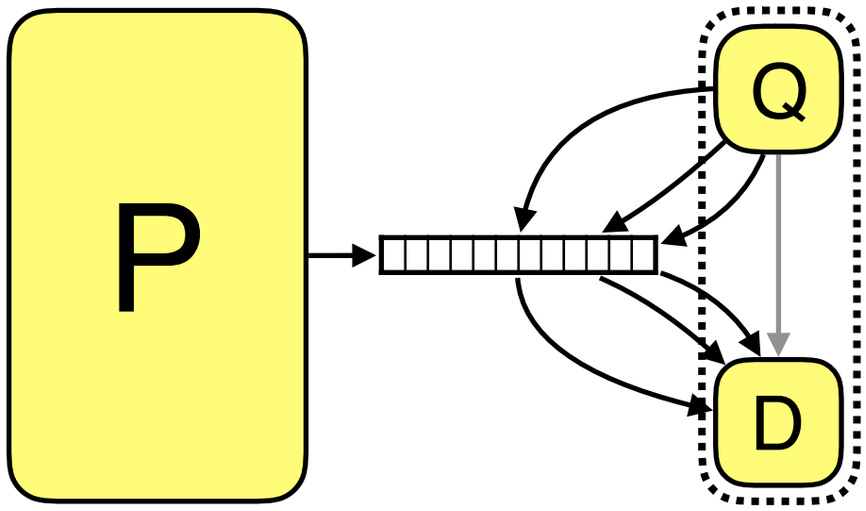
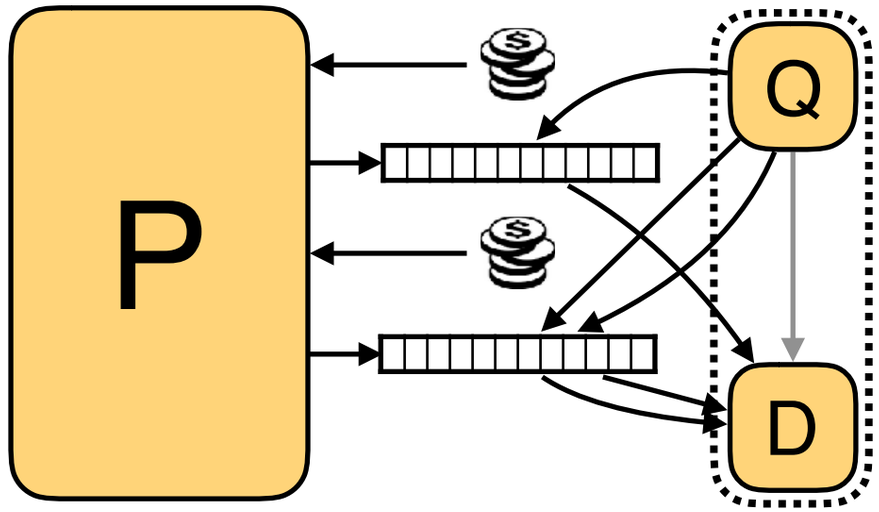
PCPs vs IOPs





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circuit SAT has 3-round IOPs
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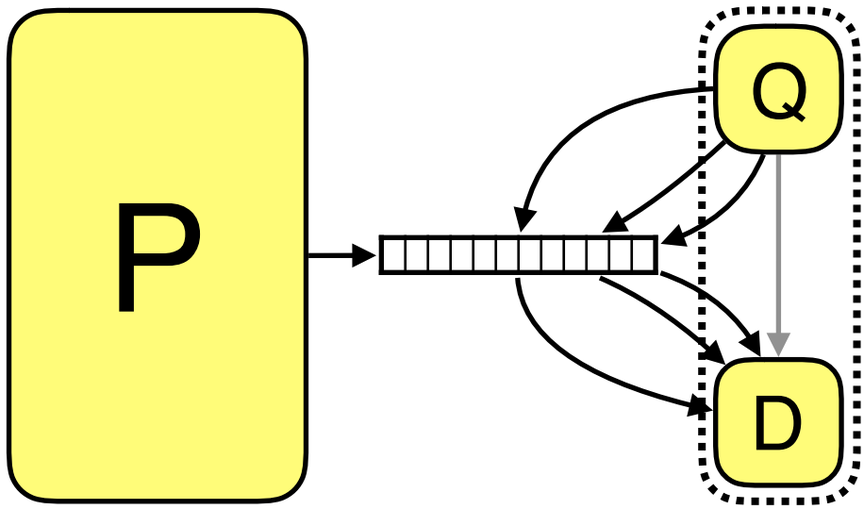
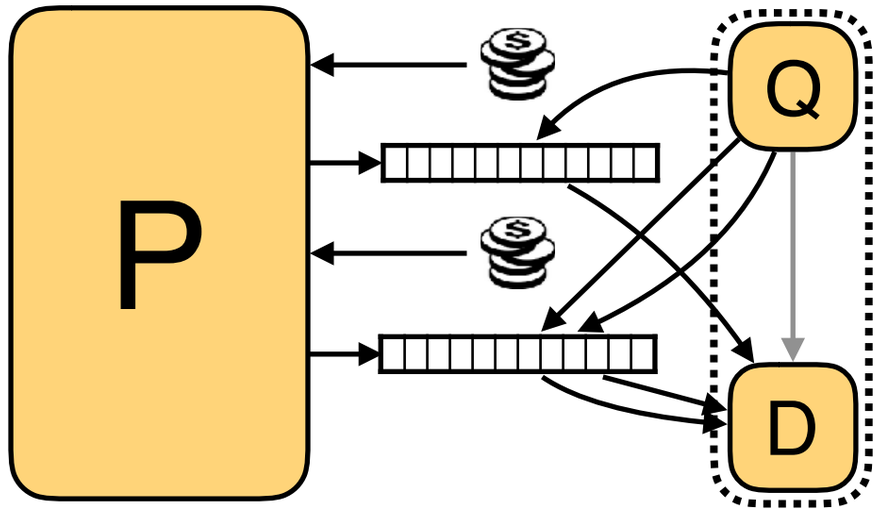
PCPs vs IOPs





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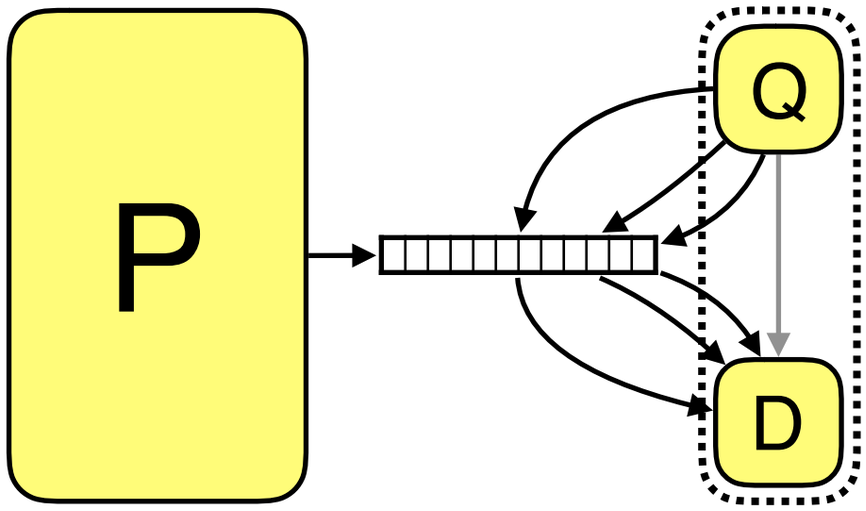
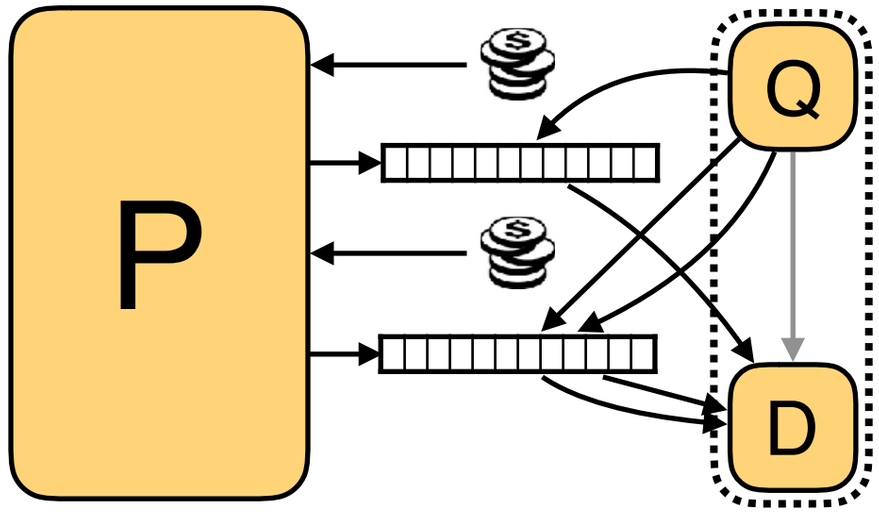
Q1: optimal round complexity?

PCPs vs IOPs





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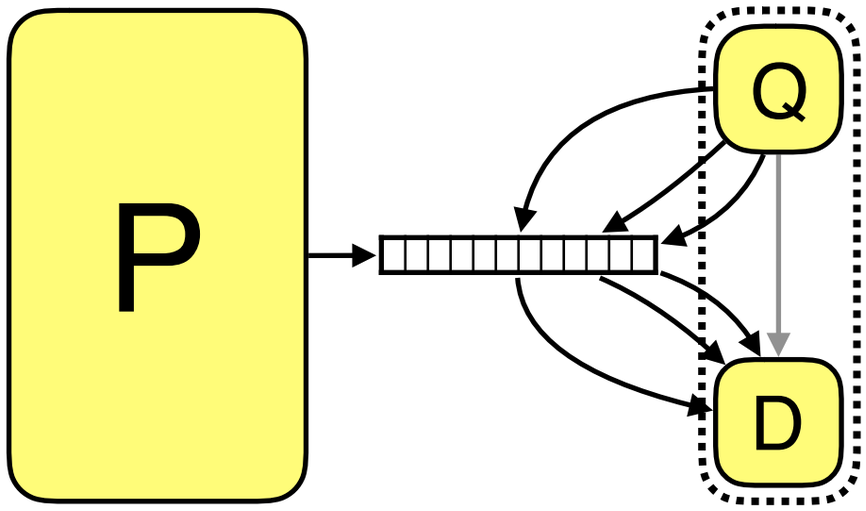
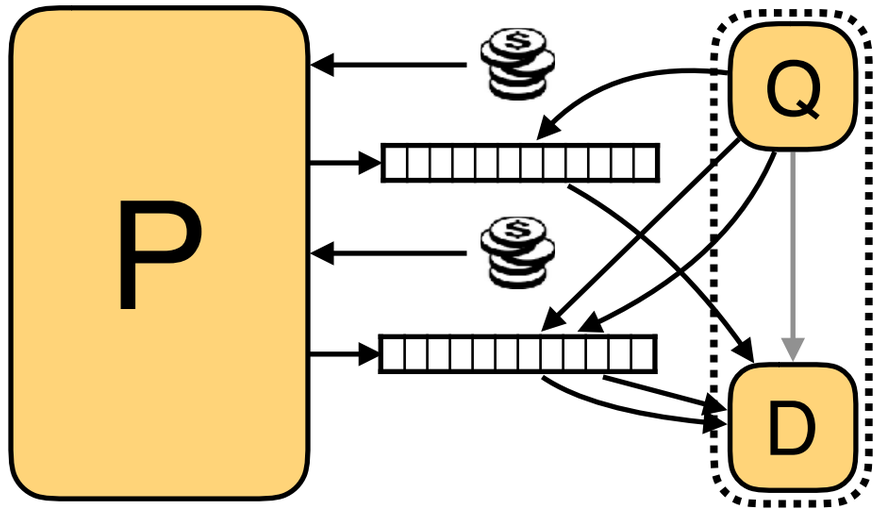
Q1: optimal round complexity?

Q2: analogs for NEXP?
(succinct circuit SAT)

PCPs vs IOPs





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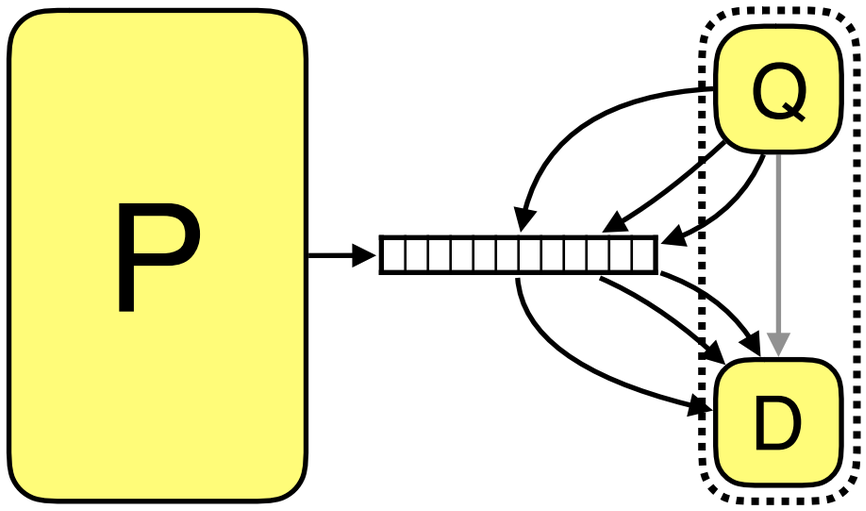
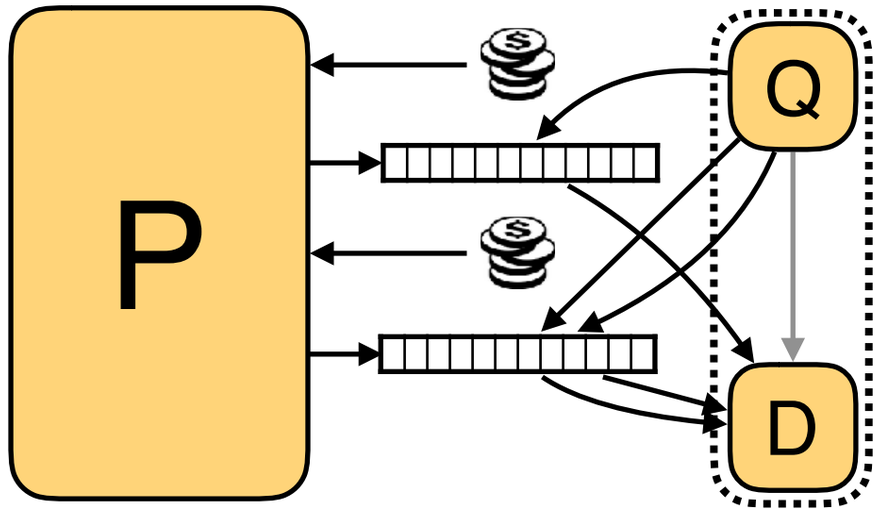
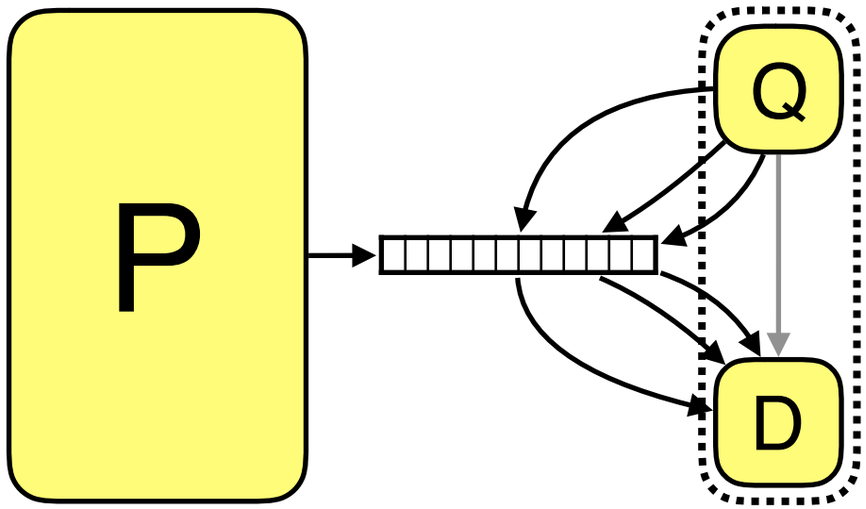
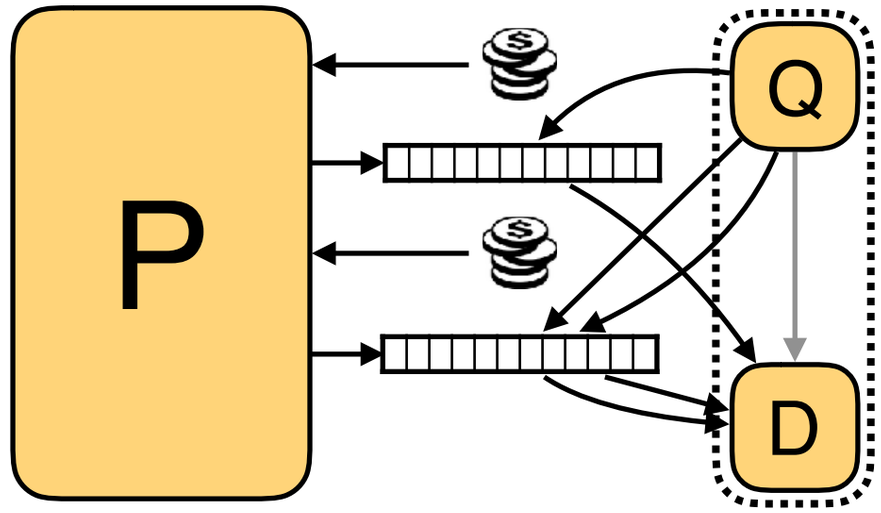




Applications to hardness of approximation?

PCPs vs IOPs





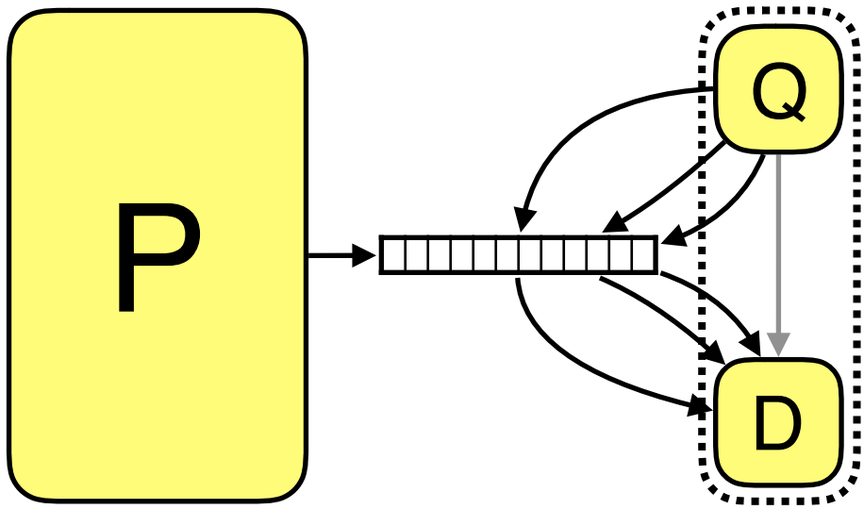
Applications to hardness of approximation?

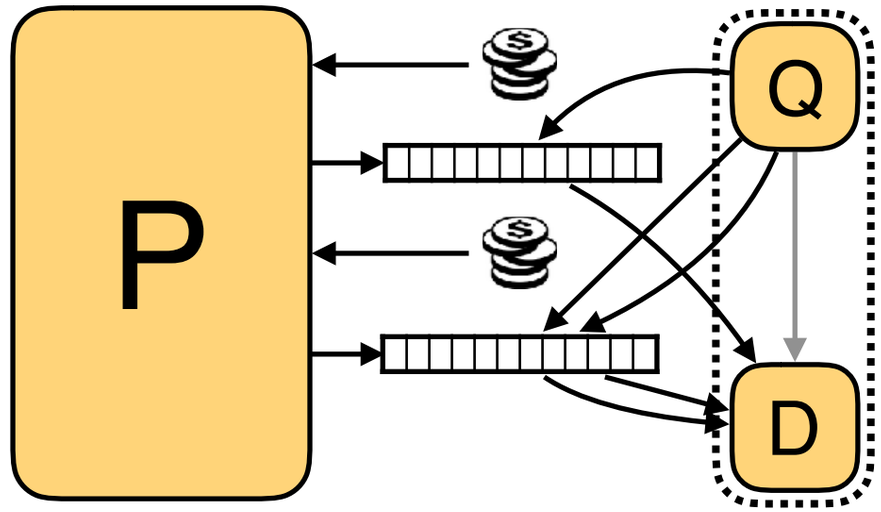
$PCP_{c,s}[r,q]_{\Sigma}$

$\updownarrow$

(c,s)-promise problem
about a q-ary CSP over Σ

PCPs vs IOPs





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↕

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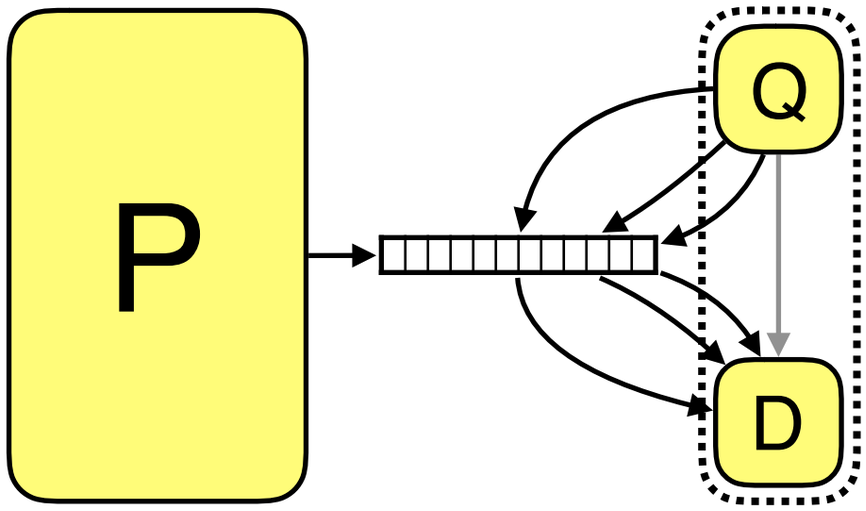
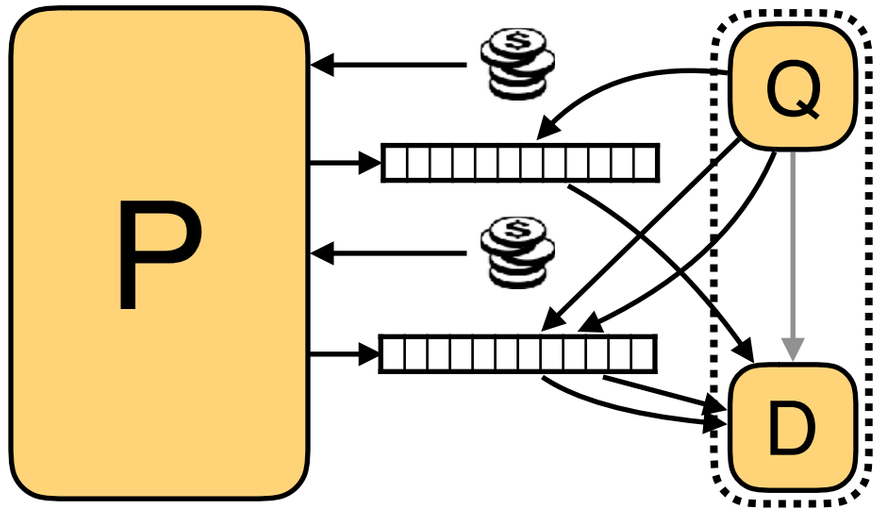
PCPs for NP-hard languages

↕

hardness of approximation for NP

PCPs vs IOPs





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???

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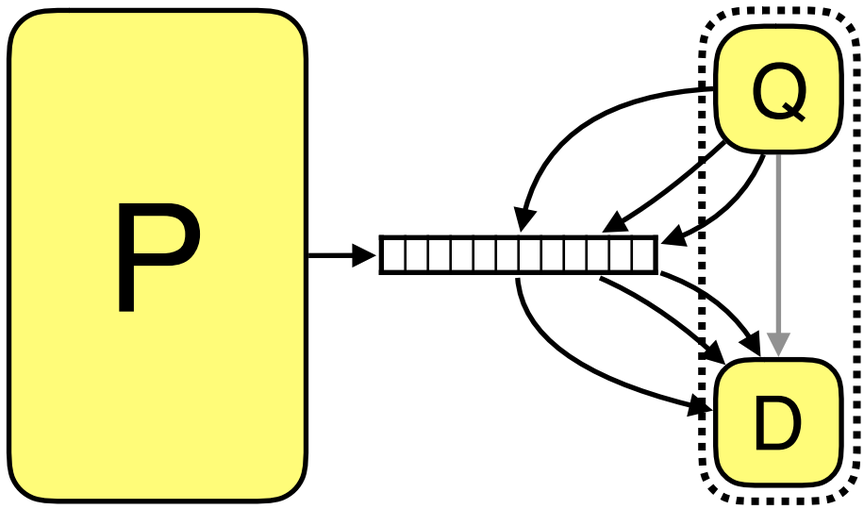

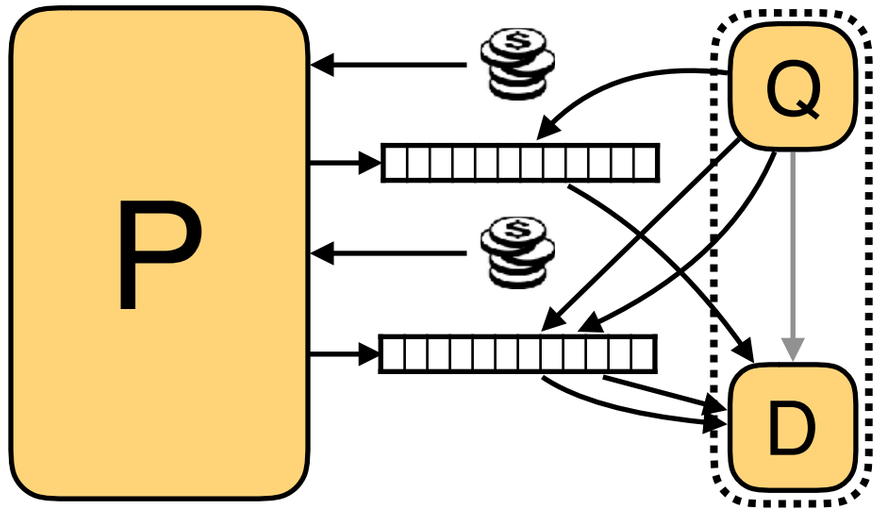

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Q: does progress on the theory of IOPs imply any interesting hardness of approximation results?

Thanks!